AD-A081 939 HAMPTON INST VA DEPT OF PHYSICS F/G 20/6
A STUDY OF TPF STRUCTURE FOR MULTIPLE OVERLAPPING OF PICOSECOND--ETC(U)
1980 D D VENABLE AFOSR-79-0036
UNCLASSIFIED AFOSR-TR-80-0205 NL

1 of 1
AD
A081939

END
DATE
FILMED
4 80
DTIC

AFOSR-TR-80-0205

A STUDY OF TPF STRUCTURE FOR MULTIPLE OVERLAPPING OF PICOSECOND LASER LIGHT PULSES

LEVEL

ADA081939

(18) AFOSR

D. D. Venable

Department of Physics
Hampton Institute
Hampton, Virginia

(15) AFOSR-79-0036

(9) Final technical rept. Dec 78 - Nov 79

(16) 2301

(17) A1

(11) 1980

(12) 73

ABSTRACT

A systematic study has been conducted to characterize the multiple overlapping of closely spaced laser pulses in a two-photon fluorescence (TPF) cell. Parametric analysis of the autocorrelation function for the amplitude of a closely spaced laser pulse train containing n-pulses has reproduced TPF spectra experimentally observed with a low-light-level electro-optics detecting system. The effects of one- and two-photon absorption has been analyzed. Theoretical predictions have been presented for peak to background ratios, number and spacings of TPF peaks, relative TPF peak intensities and fluorescent decay spectra. A design of an experimental system for generating closely spaced laser pulses has been presented. Closed form solutions have been obtained for cases in which two-photon absorption is negligible. Finally, it can be concluded that laser pulses separated by distances on the order of one picosecond can be characterized with this system.

80 3 14 77

411451

INTRODUCTION

The two-photon fluorescent, TPF, technique of Giordmaine, et al[1] is a well established method for determining laser pulse temporal durations. The author, in collaboration with J. Taboada, recently reported on a low-light-level electro-optics TPF system that incorporates a photodiode detection system.[2] In the process of developing this system, it was observed that some TPF spectra contain multiple peaks as opposed to the expected single peak. A typical oscillogram of this effect is shown in figure 1. Preliminary analysis of these data showed that the distance between closely spaced laser pulses could be determined if this process was used in a controlled manner. This project was an effort to develop a theoretical basis for such a measurement strategy.

PULSE STRUCTURES

If a laser pulse train consisting of n pulses overlaps with a mirror image of itself in a two-photon fluorescence medium, the time integrated fluorescence signal at position z in the medium will be given by equation 1. Equations 2 and 3 give the intensity distribution functions for the laser pulse train and its mirror image, $I_1(t - z/v)$ and $I_2(t + z/v)$ respectively. Table 1 contains a caption for the symbols in equations 1-3. If the two-photon absorption coefficient, β, is zero, a closed form solution exists for equation 1. The solution is given in equation 4. A laser pulse train for the case n = 3 is shown in figure 2 and a typical TPF spectrum for evenly spaced laser pulses is shown in figure 3. For the case of $\alpha = 0$ and $\beta \neq 0$ see Appendix 1. In this report all TPF spectra are normalized to give a peak signal of 3 when a single pulse overlaps with its mirror image in the TPF cell. Other parameters used to obtain the normalization constant are $\alpha = 0$, $\beta = 0$, and K = 1. The normalization constant N_3 is then

$$N_3 = \sqrt{2/\pi} / (2T_1 I_{01}^2) \qquad (5)$$

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE

READ INSTRUC
BEFORE COMPLET

1. REPORT NUMBER
AFOSR-TR- 80-0205

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG

4. TITLE *(and Subtitle)*
A STUDY OF TPF STRUCTURE FOR MULTIPLE OVERLAPPING OF PICOSECOND LASER LIGHT PULSES

5. TYPE OF REPORT & PE
Final Technical
Dec. 1978- Nov.

6. PERFORMING ORG. REPO

7. AUTHOR(s)
D. D. Venable

8. CONTRACT OR GRANT N
AFOSR-79-0036

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Department of Physics
Hampton Institute
Hampton, VA 23668

10. PROGRAM ELEMENT, PR
AREA & WORK UNIT NU
61102F 2301/A1

11. CONTROLLING OFFICE NAME AND ADDRESS
AFOSR/NP
Bolling AFB, Bldg. #410
Wash DC 20332

12. REPORT DATE
1980

13. NUMBER OF PAGES
71

14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*

15. SECURITY CLASS. *(of thi*
Unclassified

15a. DECLASSIFICATION/DO
SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*
Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*
Ultrashort laser pulses
Two-photon fluorescence
Multiple pulse structure

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

A systematic study has been conducted to characterize the multip overlapping of closely spaced laser pulses in a two-photon fluorescen (TPF) cell. Parametric analysis of the autocorrelation function for amplitude of a closely spaced laser pulse train containing n-pulses h reproduced TPF spectra experimentally observed with a low-light-level electro-optics detecting system. The effects of one- and two-photon absorption has been analyzed. Theoretical predictions have been pres

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

20. Abstract (continued)

for peak to background ratios, number and spacings of TPF peaks, relative TPF peak intensities and fluorescent decay spectra. A design of an experimental system for generating closely spaced laser-pulses has been presented. Cloud form solutions have been obtained for cases in which two-photon absorption is negligible. Finally, it can be concluded that laser pulses separated by distances on the order of one picosecond can be characterized with this system.

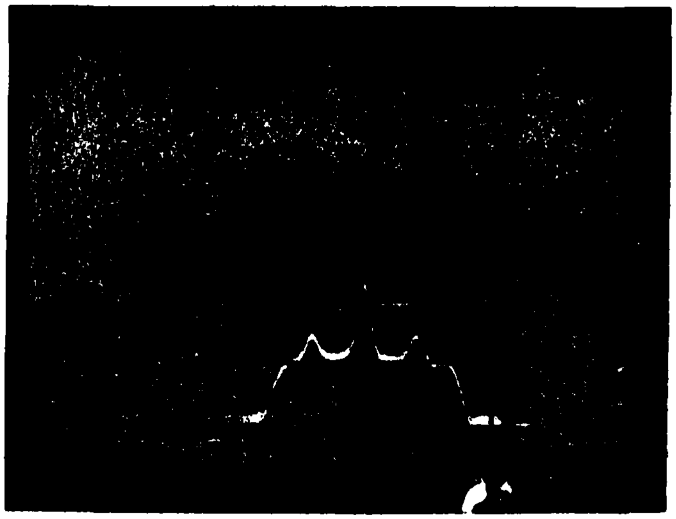


FIGURE 1

$$S_n(Z) = \int_{-\infty}^{\infty} I_1^2(t-Z/V)\,dt + \int_{-\infty}^{\infty} I_2^2(t+Z/V)\,dt + 4\int_{-\infty}^{\infty} I_1(t-Z/V)\,I_2(t+Z/V)\,dt \tag{1}$$

$$I_1(t-Z/V) = \sum_{i=1}^{n} \frac{I_{oi}\,\mathrm{EXP}\left(-\dfrac{(t-\Delta_i-Z/V)^2}{T_i^2}\right)\mathrm{EXP}\left(-\alpha(d+Z)\right)}{1+\beta\alpha^{-1}I_{oi}\,\mathrm{EXP}\left(-\dfrac{(t-\Delta_i-Z/V)^2}{T_i^2}\right)\left[1-\mathrm{EXP}\left(-\alpha(d+Z)\right)\right]} \tag{2}$$

$$I_2(t+Z/V) = \sum_{i=1}^{n} \frac{KI_{oi}\,\mathrm{EXP}\left(-\dfrac{(t-\Delta_i+Z/V)^2}{T_i^2}\right)\mathrm{EXP}\left(-\alpha(d-Z)\right)}{1+\beta\alpha^{-1}KI_{oi}\,\mathrm{EXP}\left(-\dfrac{(t-\Delta_i+Z/V)^2}{T_i^2}\right)\left[1-\mathrm{EXP}\left(-\alpha(d-Z)\right)\right]} \tag{3}$$

$$S_n(Z) \propto \sqrt{\pi}\Big(\mathrm{EXP}(-2\alpha(d+Z)) + K^2\,\mathrm{EXP}(-2\alpha(d-Z))\Big)$$

$$\times \left\{ \sum_{i=1}^{n} \sum_{j=1}^{n} \left(T_i^2 + T_j^2\right)^{-\frac{1}{2}} T_i T_j I_{oi} I_{oj} \;\mathrm{EXP}\left(- \frac{\left(\Delta_i - \Delta_j\right)^2}{T_i^2 + T_j^2} \right) \right\}$$

$$+ 4K\sqrt{\pi}\;\mathrm{EXP}(-2\alpha d) \left\{ \sum_{i=1}^{n} \sqrt{\tfrac{1}{2}}\; T_i I_{oi}^2 \,\mathrm{EXP}\left(- \frac{2(Z/V)^2}{T_i^2} \right) + C_{ij} \right\} \qquad (4)$$

Where,

$$C_{ij} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(T_i^2+T_j^2\right)^{-\frac{1}{2}} T_i T_j I_{oi} I_{oj} \left\{ \mathrm{EXP}\left(- \frac{\left(2Z/V + \left(\Delta_i - \Delta_j\right)\right)^2}{T_i^2+T_j^2} \right) + \mathrm{EXP}\left(- \frac{\left(2Z/V - \left(\Delta_i - \Delta_j\right)\right)^2}{T_i^2+T_j^2} \right) \right\}, \text{ if } n>1$$

$= 0$, if $n=1$

and $\beta = 0$.

Table 1

$S_n(Z)$	-	autocorrelation integral
I_1	-	intensity function moving in the +Z direction
I_2	-	intensity function moving in the -Z direction
Z	-	position in TPF cell
V	-	velocity of light in TPF medium
t	-	time
I_{oi}	-	peak intensity of the i^{th} Laser pulse
Δ_i	-	temporal space between the 1^{st} and i^{th} Laser pulse
α	-	single-photon absorption coefficient
β	-	two-photon absorption coefficient
T_i	-	1/e half-width of the i^{th} Laser pulse
d	-	½ dye cell width
K	-	attenuation factor for pulse moving in -Z direction
n	-	number of Laser pulses in train
θ	-	I_{01}/I_{02}
σ	-	T_1/T_2
ϵ	-	full width at half maximum (FWHM)

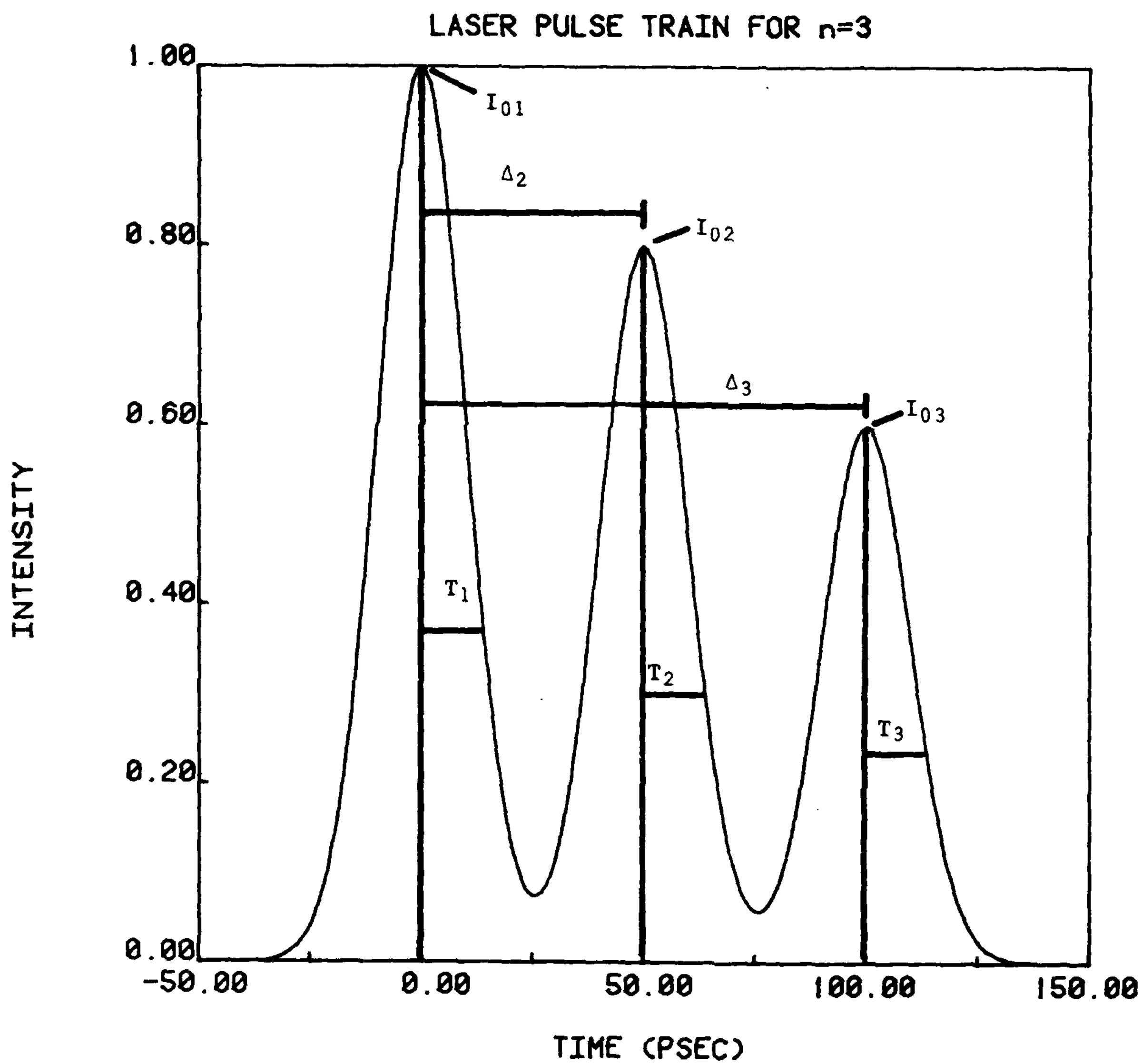

LASER PULSE TRAIN FOR n=3
INTENSITY
TIME (PSEC)
1.00
0.80
0.60
0.40
0.20
0.00
-50.00
0.00
50.00
100.00
150.00
I_{01}
I_{02}
I_{03}
Δ_2
Δ_3
T_1
T_2
T_3

FIGURE 2

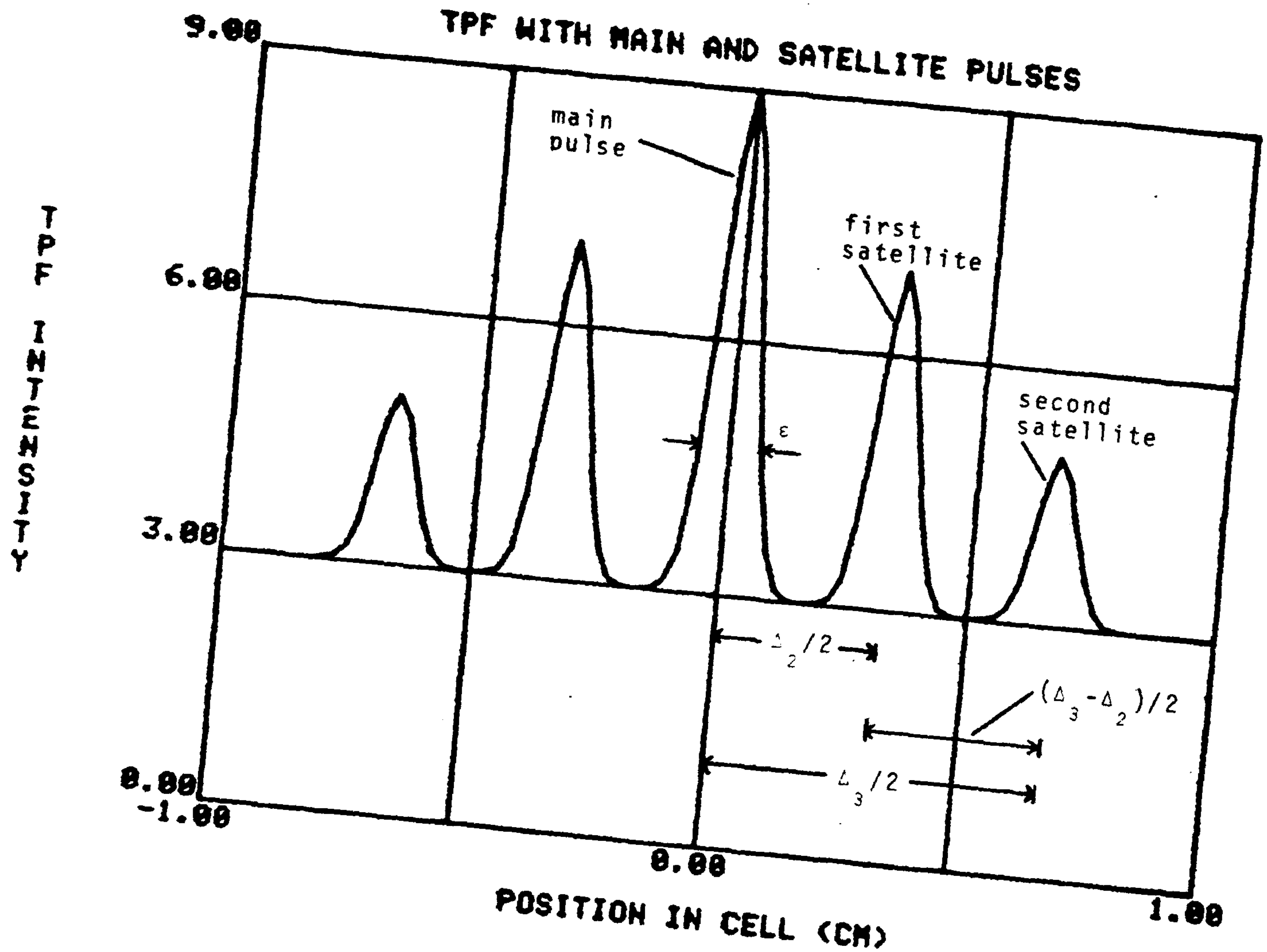

TPF WITH MAIN AND SATELLITE PULSES
TPF INTENSITY
9.00
6.00
3.00
0.00
-1.00
0.00
1.00
POSITION IN CELL (CM)
main pulse
first satellite
second satellite
ε
$\Delta_2/2$
$(\Delta_3-\Delta_2)/2$
$\Delta_3/2$

FIGURE 3

This constant is obtained by solving equation 1 for the conditions stated above then setting the results equal to 3 at $z = 0$. The full width at half-maximum of the i^{th} laser pulse is given by the relationship[3]

$$\varepsilon_i = 2T_i\sqrt{\ln 2} \qquad (6)$$

DYE CELL AND MEDIUM

In cases where the theoretical results are compared to experimental, a 1.7 cm dye cell length is used. The TPF medium is a 5×10^{-3}M solution of Rhodiman 6G in ethanol. A refractive index of 1.35 is used, thus the value used for the velocity of light in the medium is 2.22×10^{10} cm/sec. Under these conditions, pulses that are separated by distances greater than 76 ps will not have multiple overlapping within the TPF cell.

VERIFICATION OF CLOSED FORM OF $S_n(z)$

The closed form of $S_n(z)$ has been verified by comparison to numerical integration[4] of equation 1. The results of such a comparison is shown in figure 4. The parameters used in this particular comparison are $n = 2$, $I_{01} = I_{02}$, $\alpha = 0$, $K = 1$, $T_1 = T_2 = 6$ ps, $\Delta_2 = 30$ ps and $\Delta_3 = 50$ ps. The lower graph is the result of the numerical integration and the upper graph is the evaluation of equation 4.

EVEN AND UNEVEN PULSE SPACING

If the pulses in the laser pulse train are evenly spaced such that $\Delta_{i+1} - \Delta_i$ is constant for all i, then the maximum number of pulses that can occur in the TPF cell is N where

$$N = 2n - 1 \qquad (7)$$

and n is the number of pulses in the laser pulse train. Figure 5 shows TPF signals for n = 1, 2, 3, and 4 identical and evenly spaced laser pulses.

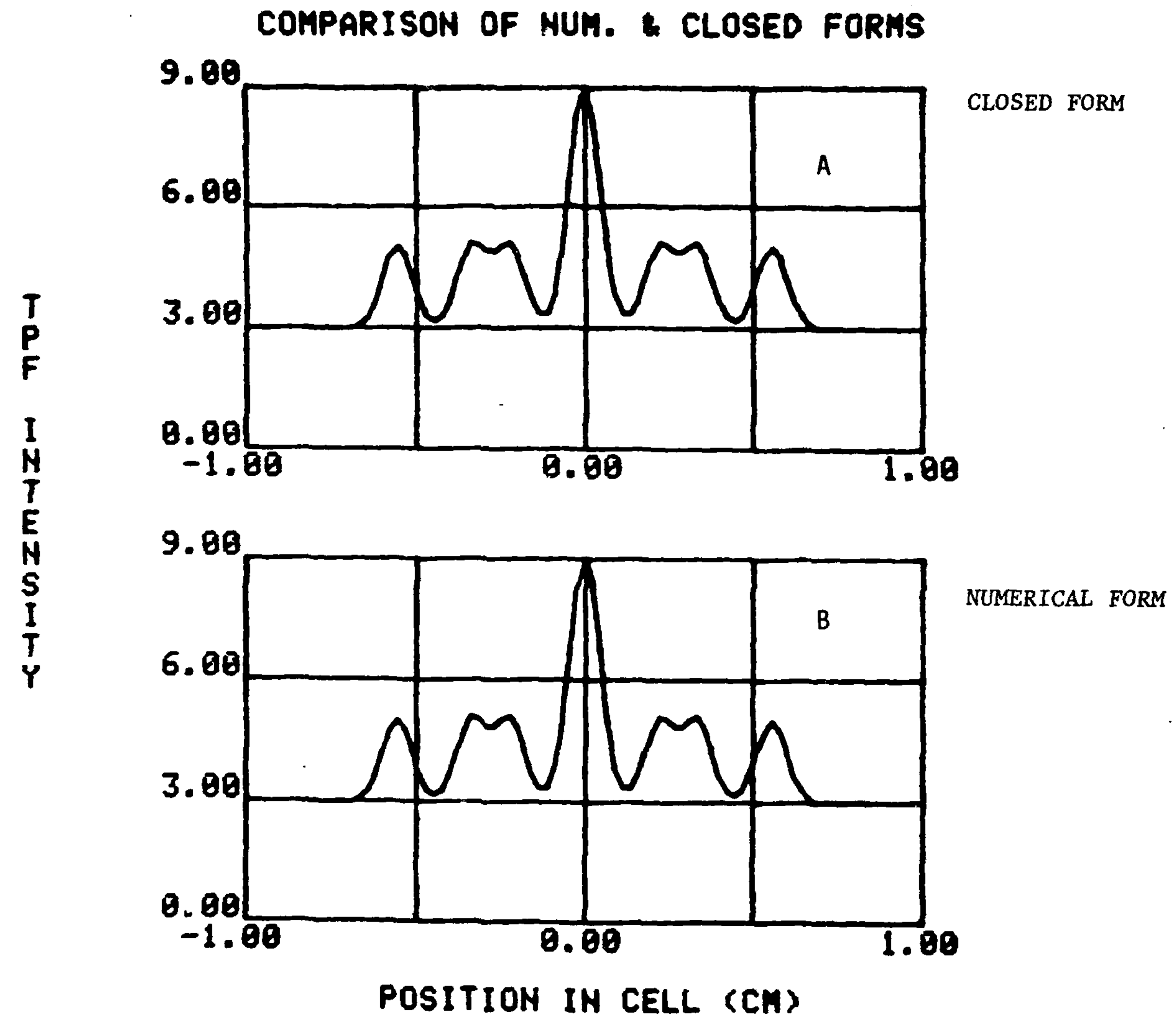

COMPARISON OF NUM. & CLOSED FORMS
9.00
6.00
3.00
0.00
-1.00
0.00
1.00
CLOSED FORM
A
NUMERICAL FORM
B
TPF INTENSITY
POSITION IN CELL (CM)

FIGURE 4

TPF FOR VARIABLE NUMBER OF EVENLY SPACED LASER PULSES

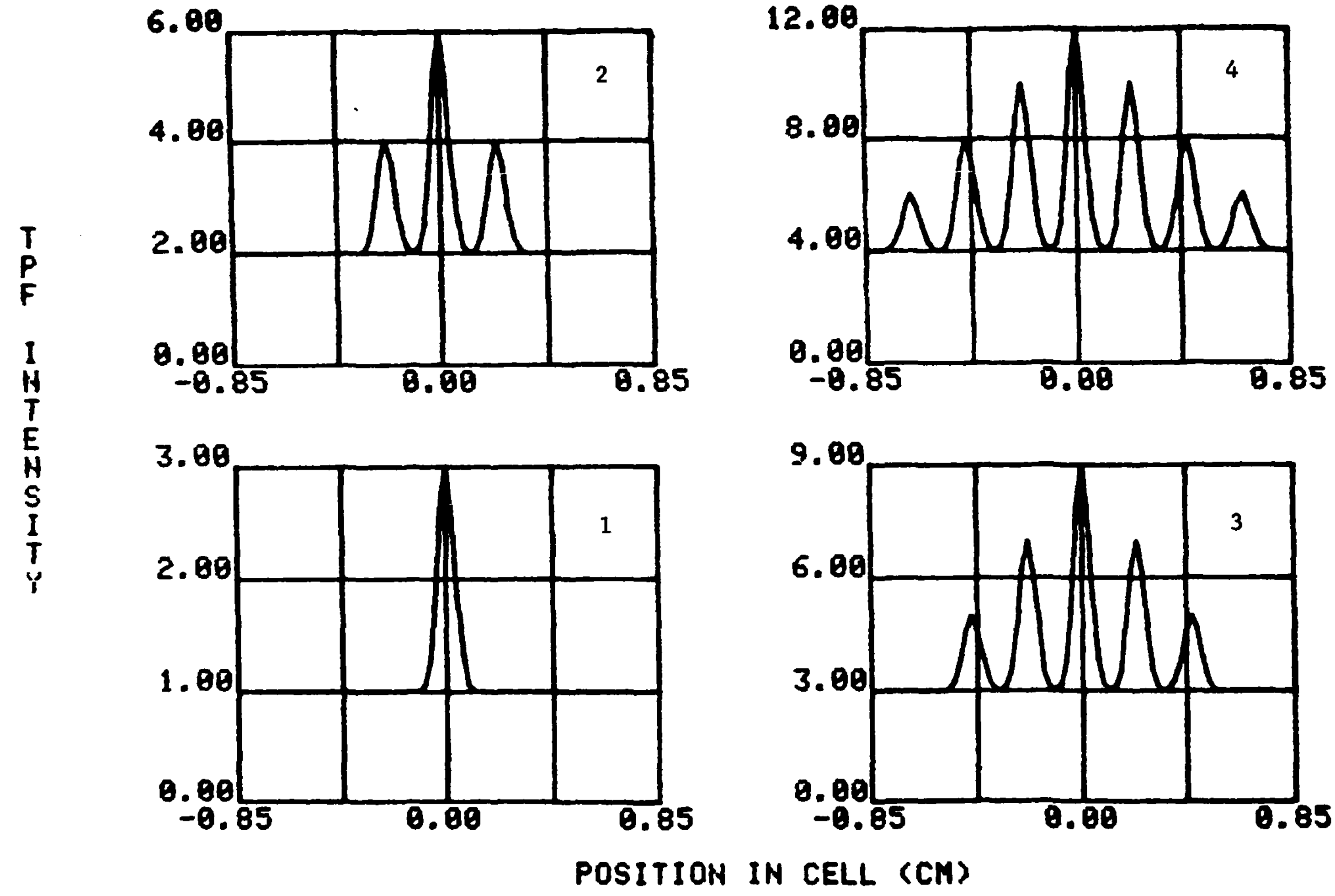


FIGURE 5

If the laser pulses are unevenly spaced, additional satellite pulses can occur within the TPF spectrum. The maximum number of pulses are found from equation 4 to be (see Appendix 2)

$$N = n^2 - n + 1. \qquad (8)$$

TPF spectra are shown in figure 6 for $n = 3$ and $\Delta_2 = 30$ ps. Δ_3 is variable and takes on values from 70 to 20 ps. Pulses may occur in the TPF cell for all values corresponding to $\pm\frac{1}{2}(\Delta_i - \Delta_j)$.

Figure 7 shows three additional TPF spectra. Graph A shows a case where the second satellite is larger than the first satellite. For this case, $n = 4$, $\Delta_2 = 40$ ps, $\Delta_3 = 80$ ps, and $\Delta_4 = 100$ ps. In B and C the same TPF spectrumis generated for two different sets of conditions. In B, $n = 6$, $\Delta_2 = 20$ ps, $\Delta_3 = 200$ ps, $\Delta_4 = 220$ ps, $\Delta_5 = 400$ ps, and $\Delta_6 = 420$ ps, and in C, $n = 2$ and $\Delta_2 = 20$ ps.

LOCATION OF TPF SATELLITE PULSES FOR EVENLY SPACED PULSE TRAIN

For an evenly spaced laser pulse train, TPF satellite pulses occur in the TPF cell at $Z/V = \pm\frac{1}{2}(\Delta_{i+1} - \Delta_i)$. Therefore, the spacing between the satellite pulses is a measure of the laser pulse separation. Figure 8 shows TPF spectra for a pulse train consisting of two identical pulses. The separation between the two pulses is variable and ranges from 80 to 10 ps. The full width at half-maximum (ϵ) of both laser pulses is 10 ps. For each case satellite pulses are located at $Z = \pm\frac{1}{2}\Delta_2 V$. Note that Δ_1 is defined to be 0.

If the pulses are evenly spaced but not identical, the same relationships hold for pulse location, but the sizes of the satellite pulses will be effected. Figure 9 shows the same case as figure 8 except that $\theta = 0.4$ and $\sigma = 0.5$. For the laser pulses $\epsilon_1 = 10$ ps and $\epsilon_2 = \sigma\epsilon_1$.

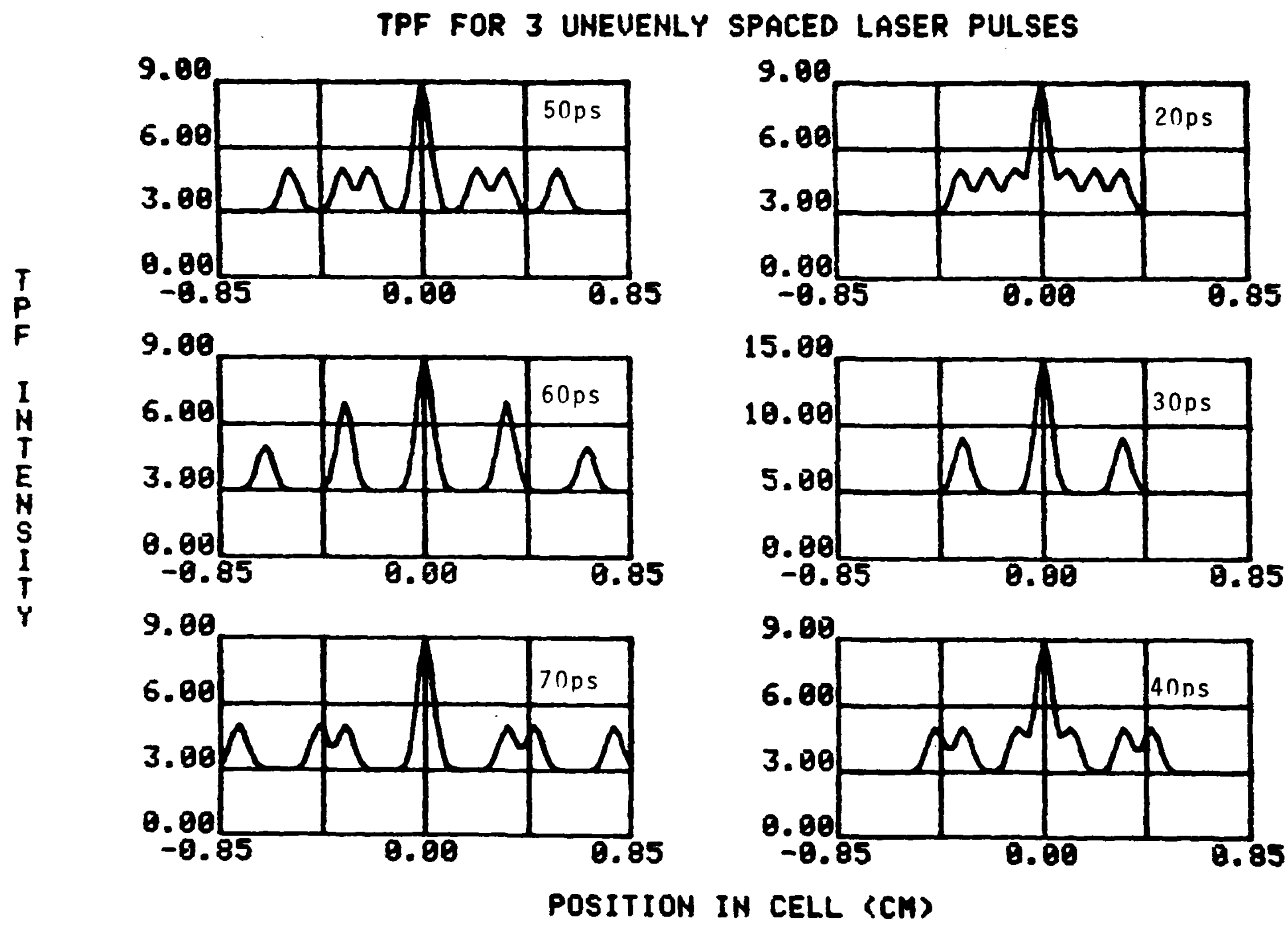

TPF FOR 3 UNEVENLY SPACED LASER PULSES
TPF INTENSITY
50ps
20ps
60ps
30ps
70ps
40ps
9.00
6.00
3.00
0.00
15.00
10.00
5.00
-0.85
0.00
0.85
POSITION IN CELL (CM)

FIGURE 6

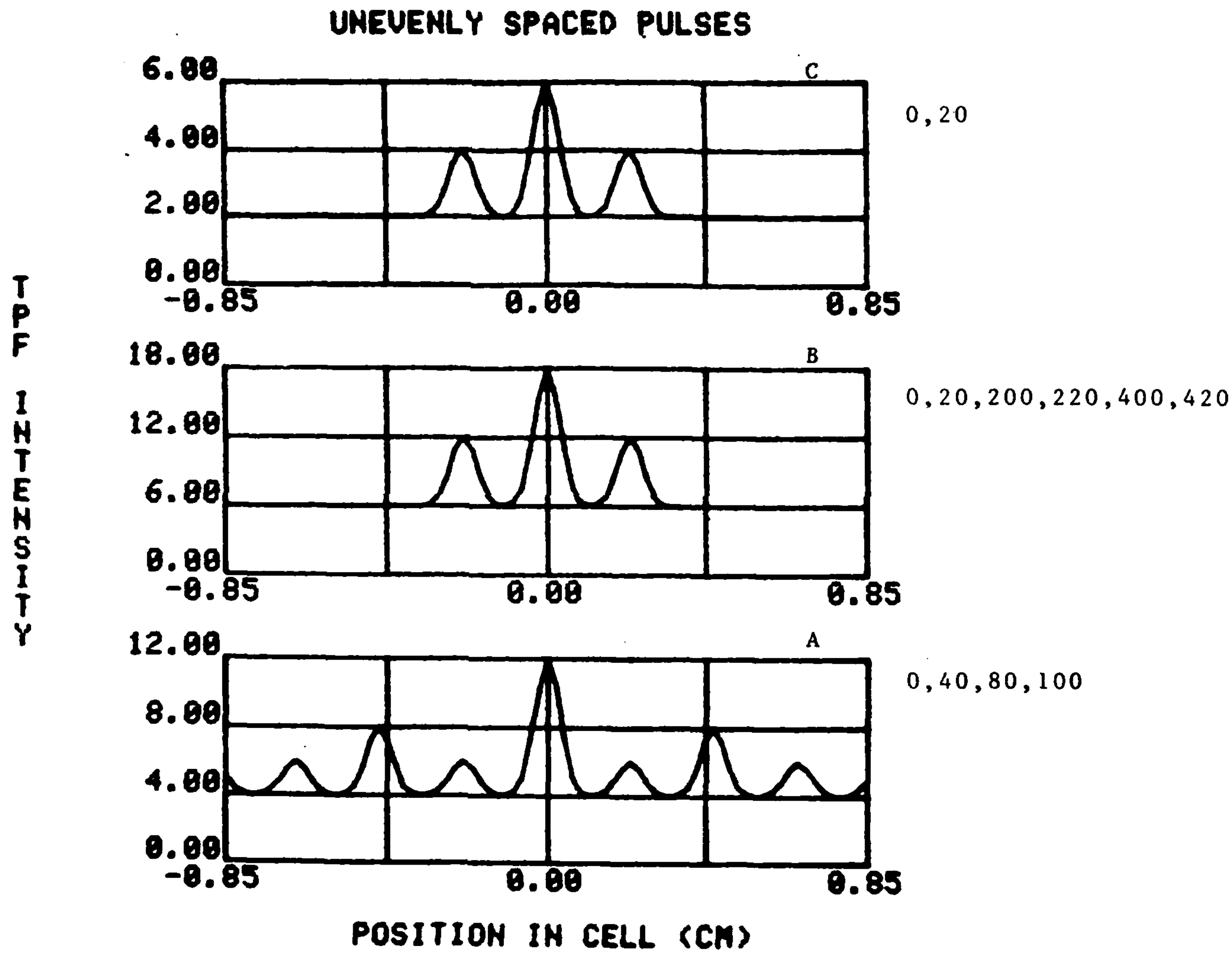

UNEVENLY SPACED PULSES
TPF INTENSITY
6.00
4.00
2.00
0.00
-0.85
0.00
0.85
C
0,20
18.00
12.00
6.00
0.00
-0.85
0.00
0.85
B
0,20,200,220,400,420
12.00
8.00
4.00
0.00
-0.85
0.00
0.85
A
0,40,80,100
POSITION IN CELL (CM)

FIGURE 7

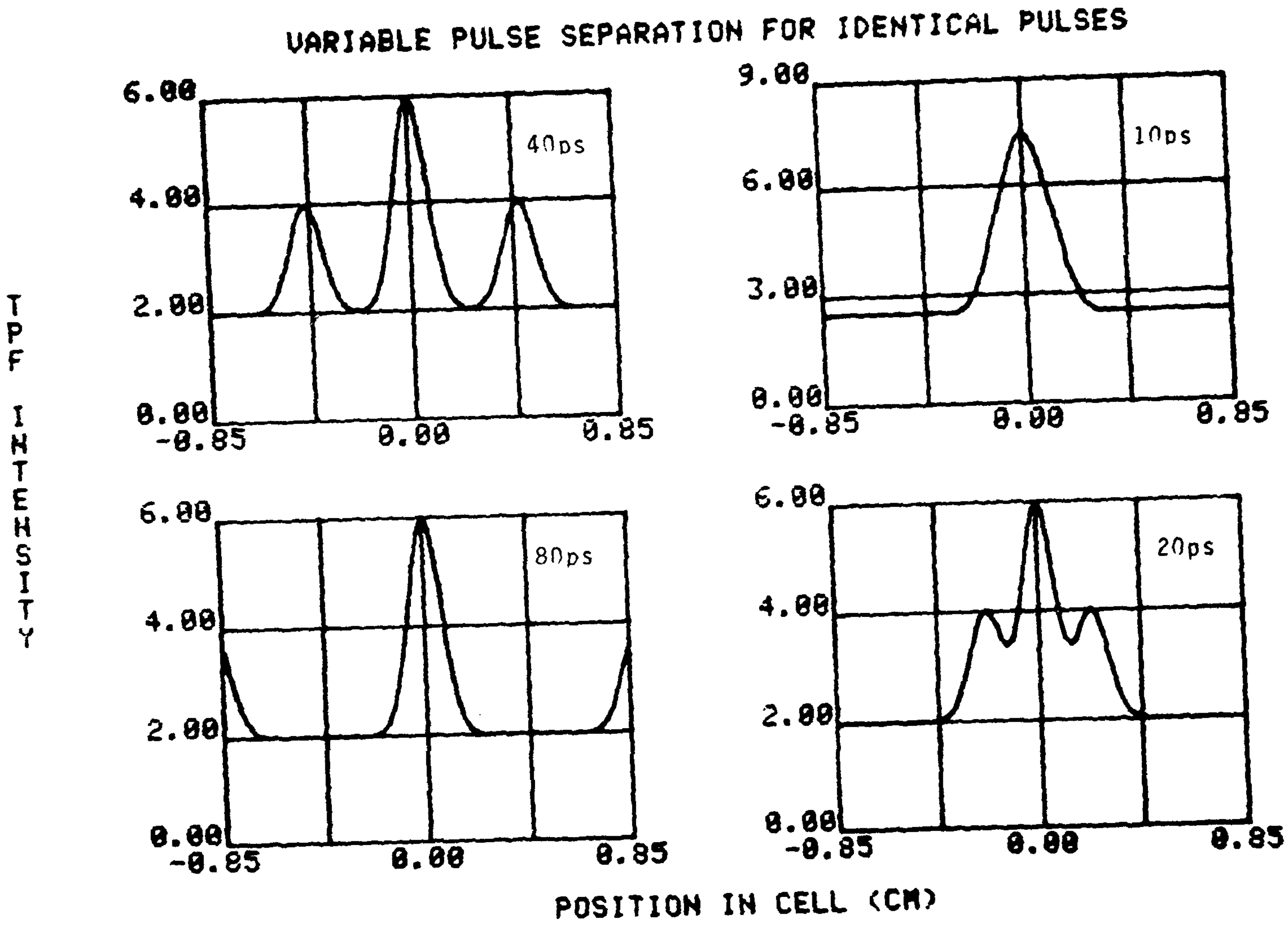

VARIABLE PULSE SEPARATION FOR IDENTICAL PULSES
TPF INTENSITY
40ps
10ps
80ps
20ps
6.00
4.00
2.00
0.00
9.00
3.00
-0.85
0.00
0.85
POSITION IN CELL (CM)

FIGURE 8

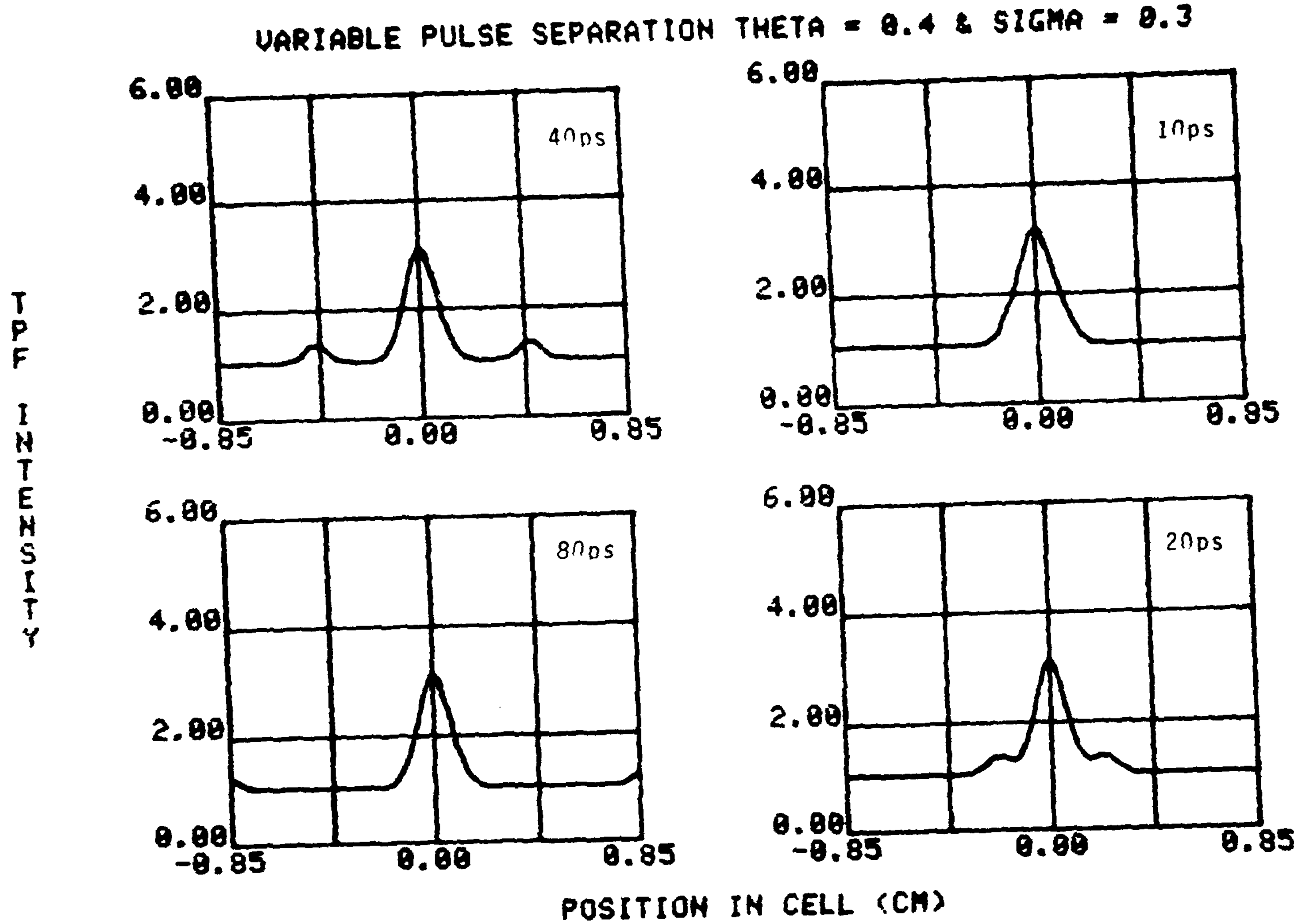

VARIABLE PULSE SEPARATION THETA = 0.4 & SIGMA = 0.3
TPF INTENSITY
POSITION IN CELL (CM)
40ps
10ps
80ps
20ps
6.00
4.00
2.00
0.00
-0.85
0.00
0.85

FIGURE 9

HEIGHT OF MAIN PULSE vs HEIGHT OF SATELLITE PULSES

The ratios of the height of the main pulse to the height of satellite pulses have been shown to depend on the number of laser pulses overlapping in the TPF cell. Let H_m be the height of the main peak above the background level and let H_s^i be the height of the i^{th} satellite peak. For the case of two pulses overlapping in the TPF cell one satellite pulse occurs. (Symmetrical pulses are seen to the left and right of the main pulse).

For this case, closed form calculations have been used to show that the ratio H_m/H_s^1 has a minimum value of 2. (See Appendix 3). The minimum occurs when both laser pulses are identical. Let $T_2/T_1 = \sigma$ and $I_{02}/I_{01} = \theta$ when θ and/or σ are less than one, the ratio H_m/H_s^1 is always greater than 2.

Figure 10 shows a laser pulse train and the corresponding definitions for θ and σ. Figure 11 shows the ratio H_m/H_s^1 for various values of θ and σ.

When N identical laser pulses which are separated by equal spaces overlap in the TPF cell, the ratio becomes

$$H_m/H_s^1 = n/(n-1). \tag{9}$$

Figure 12 shows a series of overlaps for 2, 3, 4, 5, 6, and 10 pulses. The separation of the pulses is such that only the first and second overlap occur within the TPF cell. The ratios for additional satellites are given by

$$H_m/H_s^i = n/(n-i). \tag{10}$$

EFFECTS OF α AND K

Single-photon absorption in the TPF medium causes deviation in the TPF spectrum from the ideal case. A series of TPF spectra for various

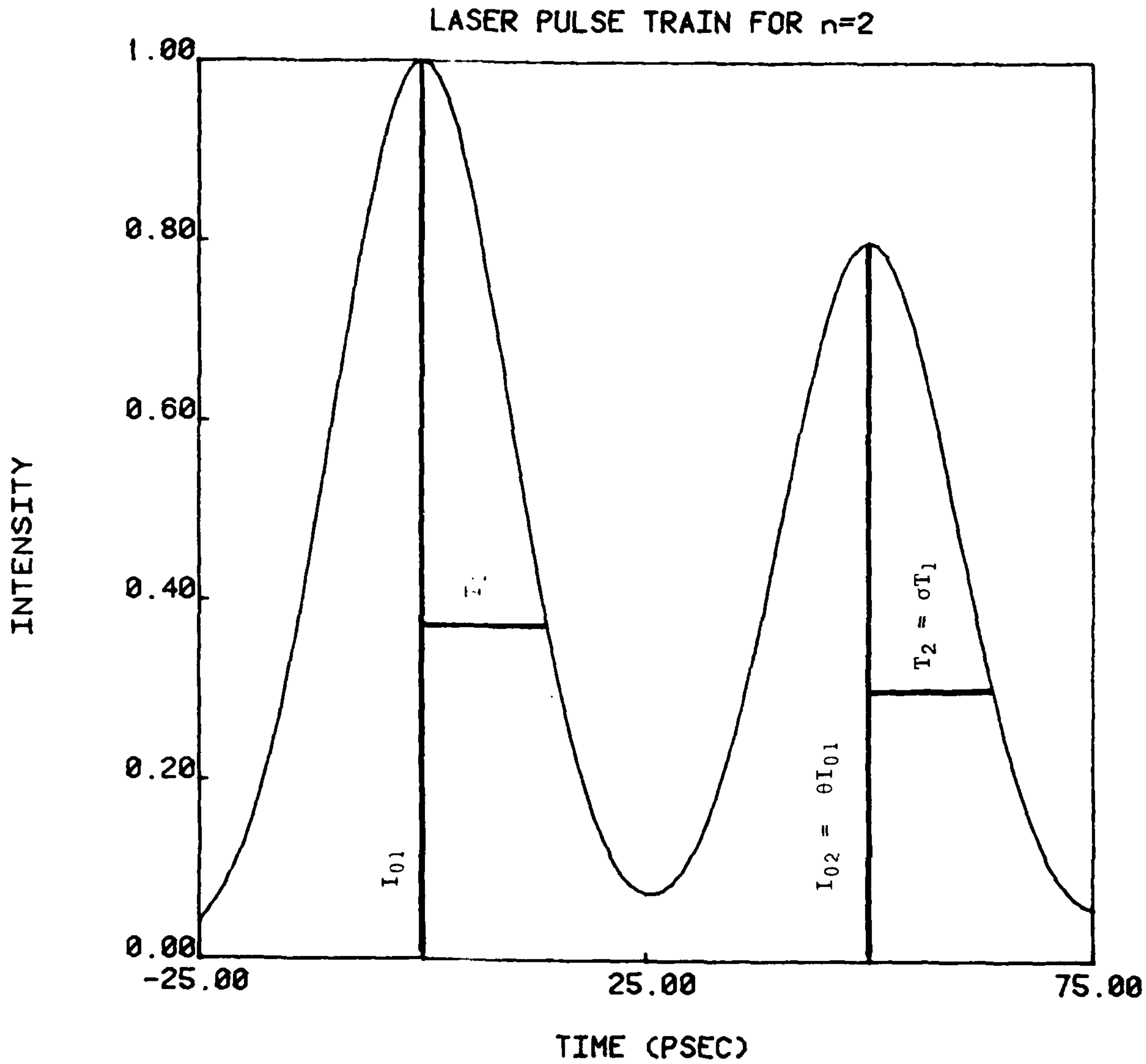

LASER PULSE TRAIN FOR n=2
INTENSITY
1.00
0.80
0.60
0.40
0.20
0.00
-25.00
25.00
75.00
TIME (PSEC)
I_{01}
$I_{02} = \theta I_{01}$
$T_2 = \sigma T_1$

FIGURE 10

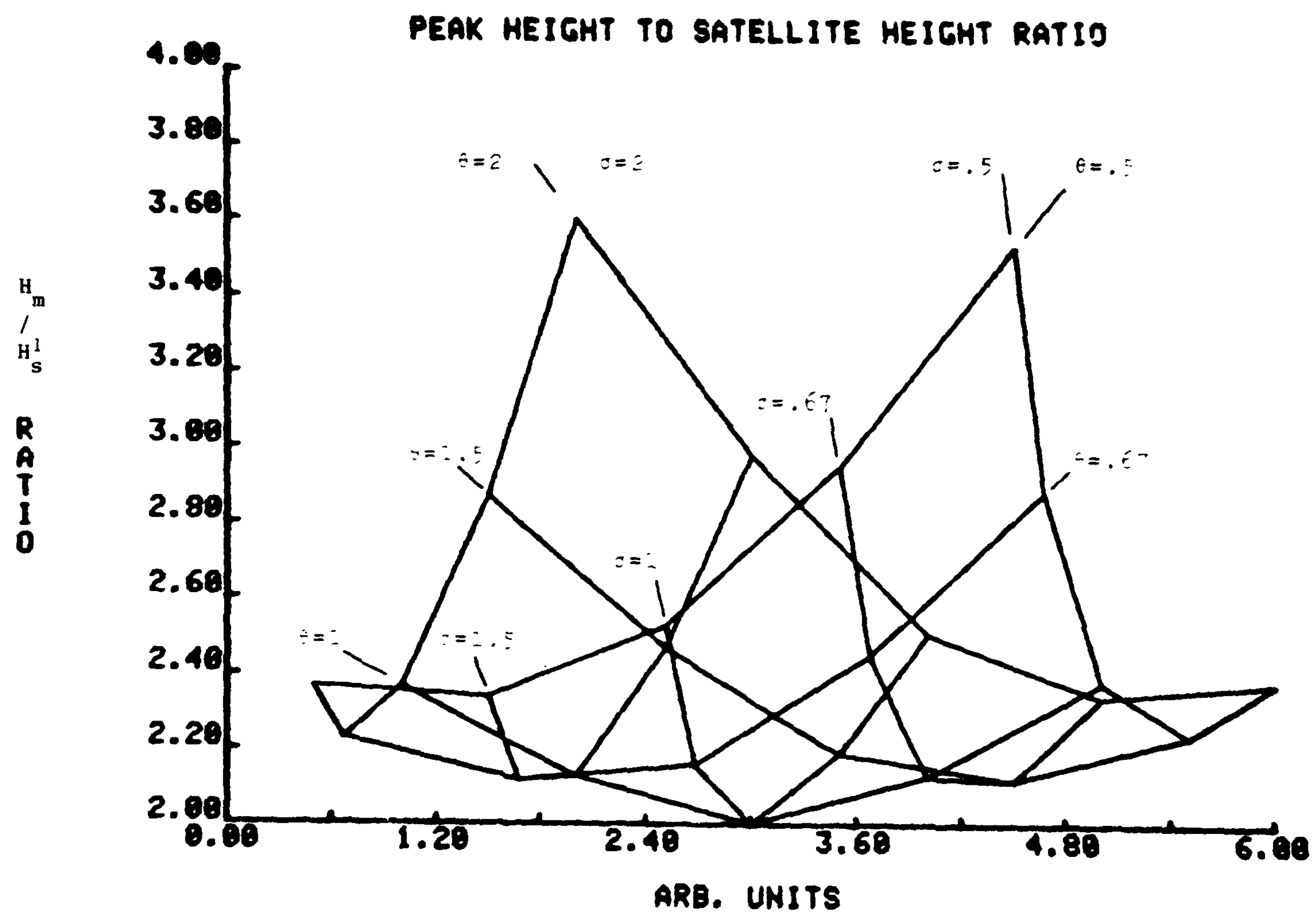

PEAK HEIGHT TO SATELLITE HEIGHT RATIO
4.00
3.80
3.60
3.40
3.20
3.00
2.80
2.60
2.40
2.20
2.00
0.00
1.20
2.40
3.60
4.80
6.00
ARB. UNITS
H_m / H_s^1
RATIO
θ=2
σ=2
σ=.5
θ=.5
σ=.67
θ=.67
σ=1.5
σ=1
θ=1
σ=1.5

FIGURE 11

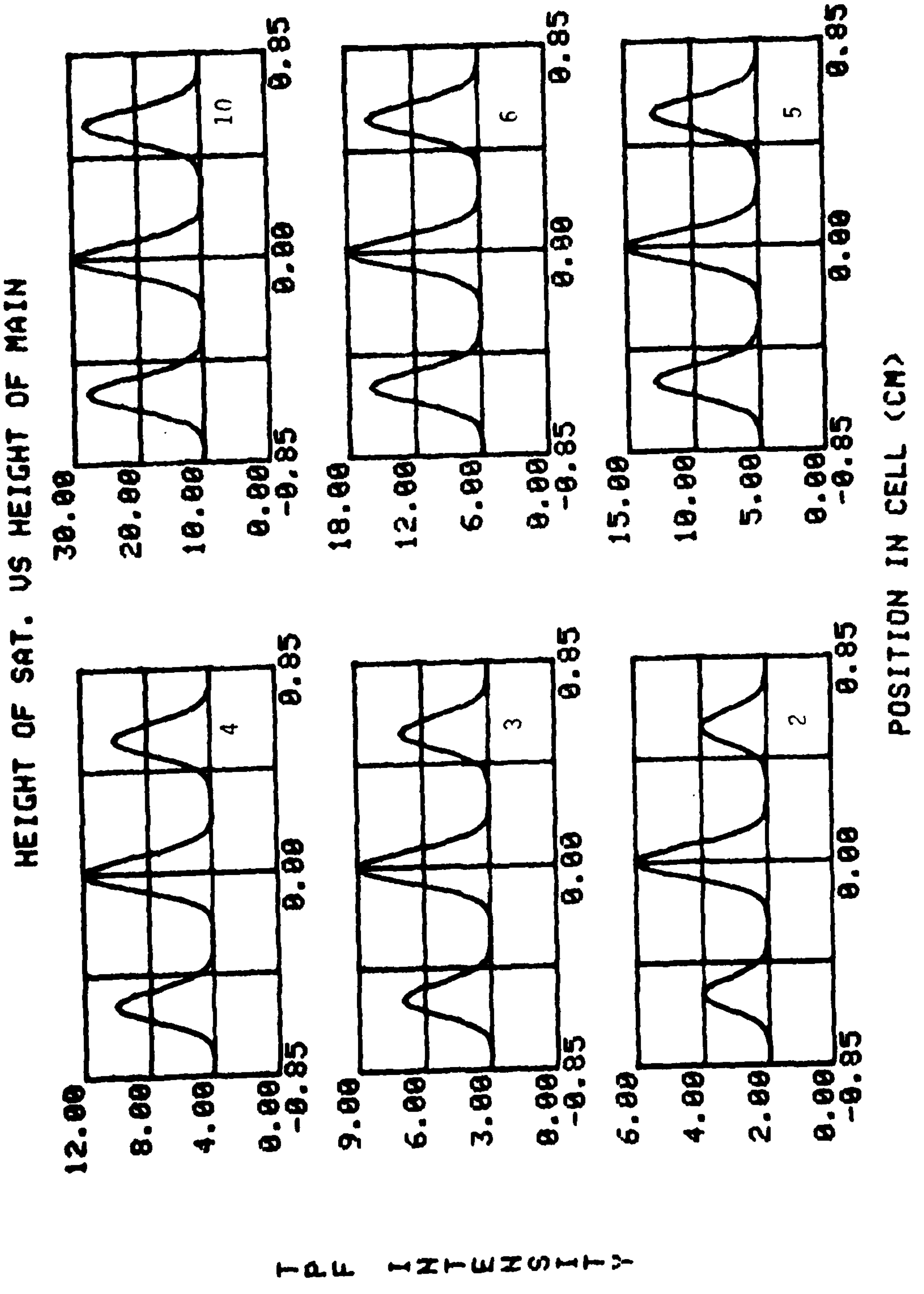

HEIGHT OF SAT. VS HEIGHT OF MAIN
TPF INTENSITY
POSITION IN CELL (CM)
10
6
5
4
3
2
30.00
20.00
10.00
18.00
12.00
6.00
15.00
5.00
12.00
8.00
4.00
9.00
3.00
2.00
0.00
-0.85
0.00
0.85

FIGURE 12

values of α is shown in figure 13. The peak to background ratio is less than 3:1 for all cases. For single pulses, the maximum value for the ratio is given by R' where (see Appendix 4)

$$R' = \frac{(1 + K^2 + 4K)e^{-2\alpha d}}{1 + K^2 e^{-4\alpha d}}, \tag{11}$$

here R' is the ratio S(0)/S(-d). All graphs in figure 13 are evaluated using K = 1. Figure 14 shows variations in the TPF spectrum when $\alpha = 0$ for various values of K. For all cases the peak to background ratio is given by equation 11. In addition, analysis of the full width at half-maximum (ε) of the TPF signal shows the ε weakly depends on the value of K, at K = 1, ε = 10.0 ps but at K = 0.5, ε = 11.1 ps. The actual value of ε for the laser pulses is 10 ps for all cases.

FWHM Vs σ AND θ

If the separation between laser pulses is large, only the main pulse might be observed in the TPF cell. Care must be taken in interpreting the full width half-maximum of the TPF trace. Figure 15 shows variations in FWHM for various values of σ and θ for two laser pulses overlapping in the TPF cell. For these cases, I_{01} = 1 GW/cm^2 and T_1 = 6 psec.

OPTICAL PULSE MULTIPLER

The purpose of the optical pulse multiplier shown in figure 16 is to use a single laser pulse to generate a train of n laser pulses all with equal amplitudes. To minimize lateral displacement of the laser beam upon transmission, the surfaces S_1, S_2 . . . S_j are all pellicle beam splitters. If the number of pulses in the pulse train is n, then the number of beam splitters is j when j = 2n. If the beam

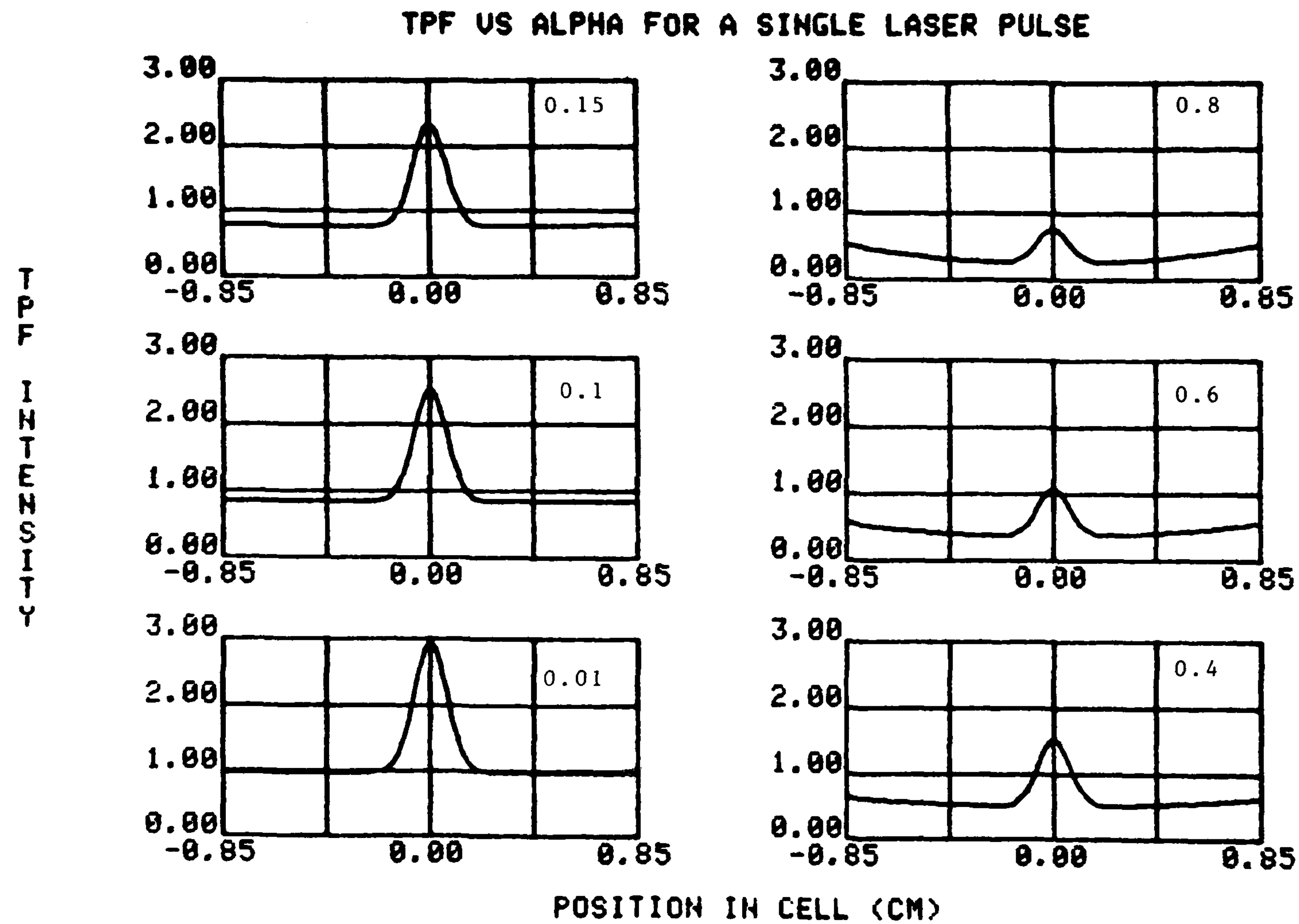
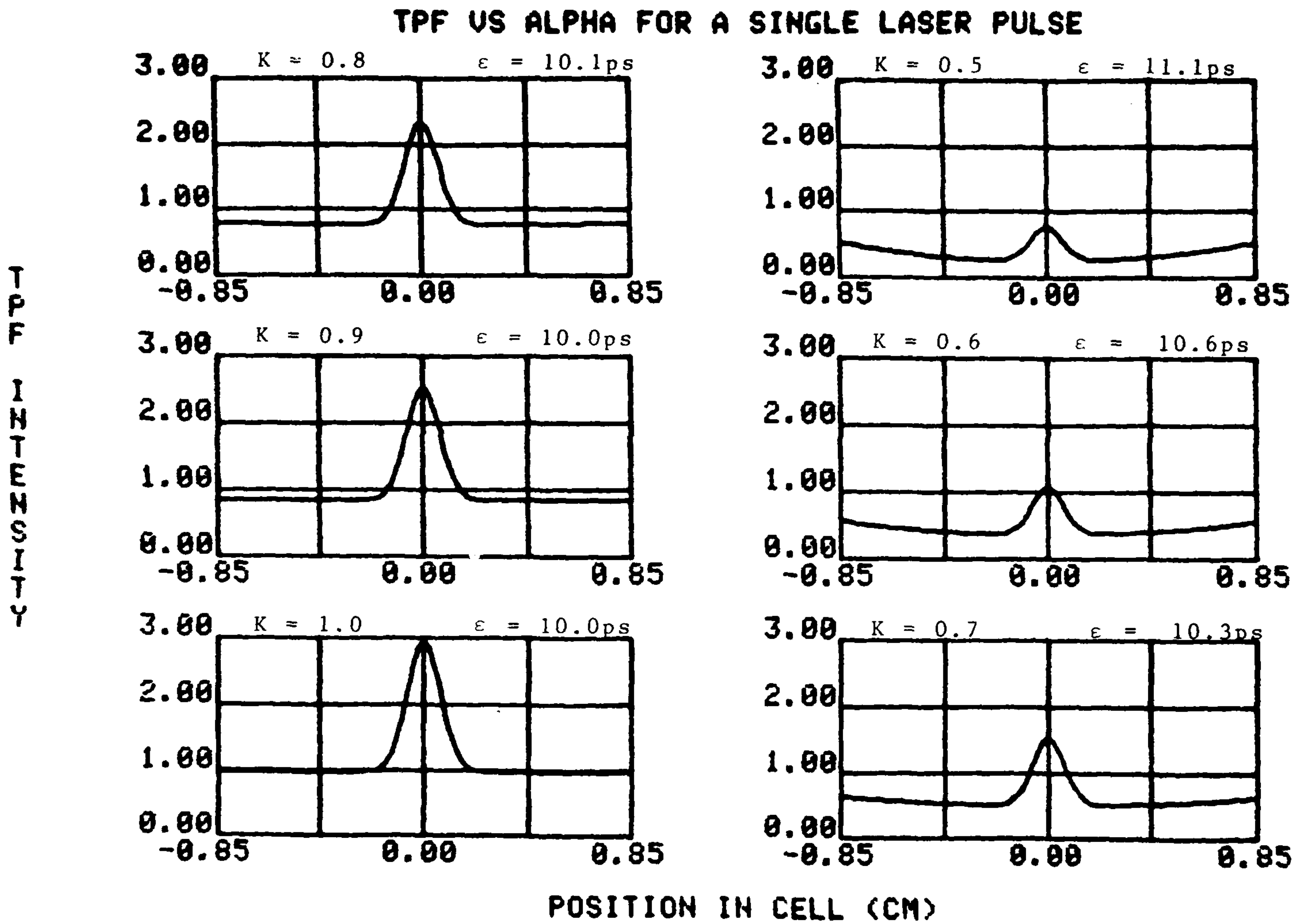

TPF VS ALPHA FOR A SINGLE LASER PULSE
TPF INTENSITY
3.00
2.00
1.00
0.00
-0.85
0.00
0.85
0.15
0.1
0.01
0.8
0.6
0.4
POSITION IN CELL (CM)

FIGURE 13



TPF VS ALPHA FOR A SINGLE LASER PULSE
K = 0.8
ε = 10.1ps
K = 0.9
ε = 10.0ps
K = 1.0
ε = 10.0ps
K = 0.5
ε = 11.1ps
K = 0.6
ε = 10.6ps
K = 0.7
ε = 10.3ps
3.00
2.00
1.00
0.00
-0.85
0.00
0.85
TPF INTENSITY
POSITION IN CELL (CM)

FIGURE 14

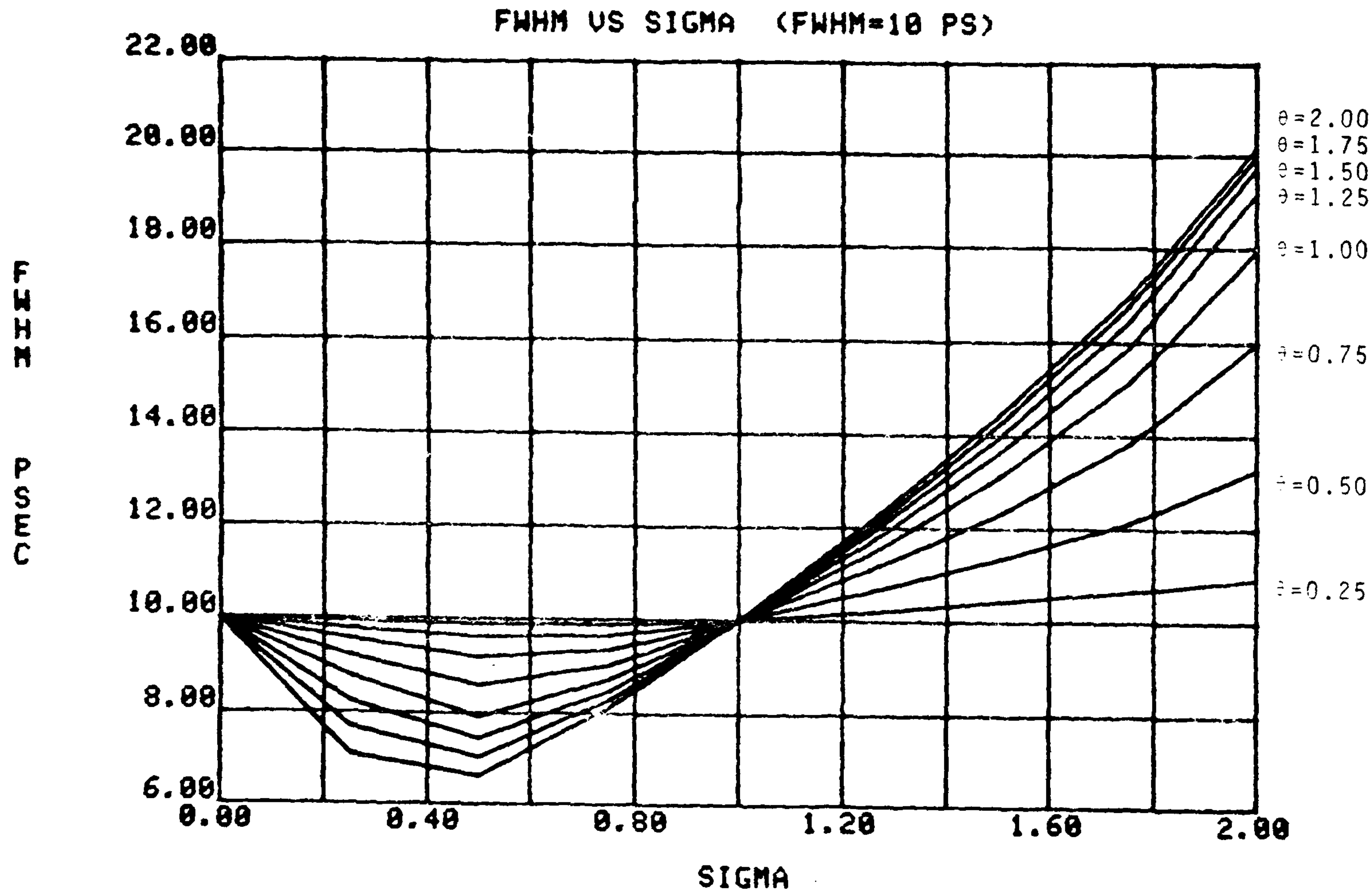

FWHM VS SIGMA (FWHM=10 PS)
22.00
20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
FWHM PSEC
0.00
0.40
0.80
1.20
1.60
2.00
SIGMA
θ=2.00
θ=1.75
θ=1.50
θ=1.25
θ=1.00
θ=0.75
θ=0.50
θ=0.25

FIGURE 15

splitters are identical the peak intensities of all pulses in the trains will be equal, and given by I_{oi} where

$$I_{oi} = r^2(1 - r)^{n-1} I_o, \tag{12}$$

where I_o is the peak intensity of the incident pulse and r is the reflectivity of the beam splitter. Equation 12 maximizes when

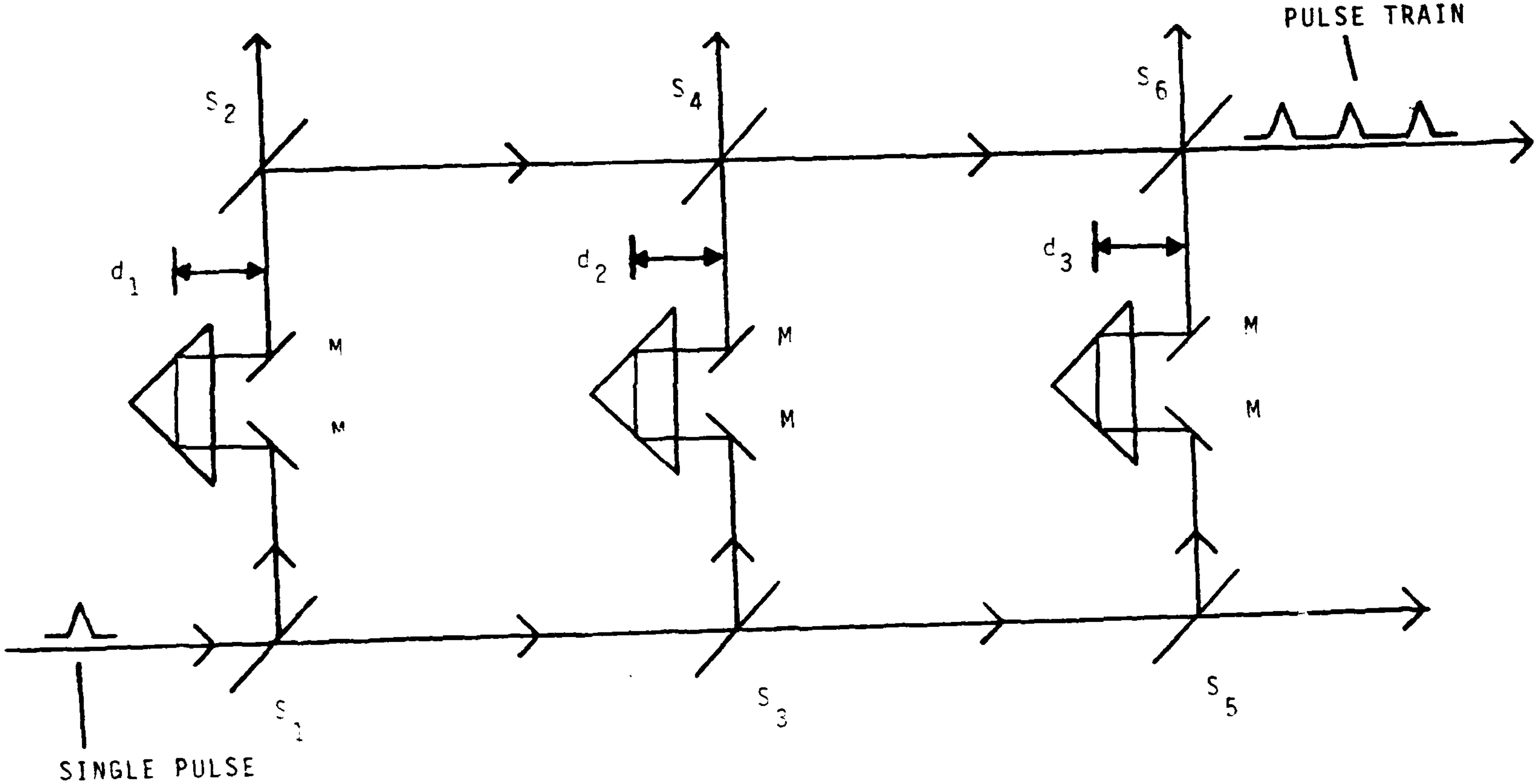
$$r = \frac{2}{n + 1} . \tag{13}$$

Therefore, the system can be optimized when designed to produce a specific number of pulses. The intensity of the pulses will be given by

$$I_{oi} = \frac{4}{n^2 -1} \left(\frac{n - 1}{n + 1} \right)^n I_o . \tag{14}$$

If the system is to be used to generate a variable number of pulses, a value of r should be selected to give an optimum value of I_{oi} for the maximum number of pulses n to be generated. Smaller numbers of pulses, K can be generated by placing an opaque object in the lower beam path after the K^{th} beam splitter in that path. The intensity of the pulses in any train will be still given by equation 14, where n is the maximum number of pulses that can be generated.

Highly accurate placement of the beam splitters is not required since the differences in optical paths of the first and i^{th} pulse in the train is given by $2(d_1 - d_i)$. Micrometer type adjustments capable of controlling each optical delay to within 0.1 mm will give a resolution in pulse separation better than 1 ps. In addition, space requirements for the optical pulse multiplier will be minimized if the system is constructed with vertical optical paths.



OPTICAL PULSE MULTIPLIER

FIGURE 16

TEMPORAL SEPARATION OF PULSES FOR PULSE MULTIPLIER

The separation between the first pulse and i^{th} pulse is given by Δ_i where

$$\Delta_i = 2(d_i - d_1)/C, \; i > 1 \tag{15}$$

and C is the speed of light in vacuum. A micrometer with 0.1 mm accuracy and a range of 15 cm would be sufficient to generate pulses with separations from 1 ps to 1 ns.

To generate the desired pulse separation, the steps listed below should be followed. It is assumed that n pulses are to be generated and spacing between successive pulses in the train is to be a constant ($\Delta_i = (i - 1)\,\Delta_2$). In addition, $\Delta_2 V$ must be less than the length of the TPF cell where V is velocity of light in the TPF medium.

1. Adjust and lock d_1 near its minimum value.

2. Block the beam after the beam splitter that reflects light to the second optical delay.

3. Decrease d_2 until a satellite pulse occurs in the TPF cell and is separated from the main pulse by $V\,\Delta_2/2$. If the background is subtracted out the ratio R of the main pulse to the satellite pulse will be 2:1.

4. Relocate the optical beam block to allow light to be reflected to one additional optical delay, d_i.

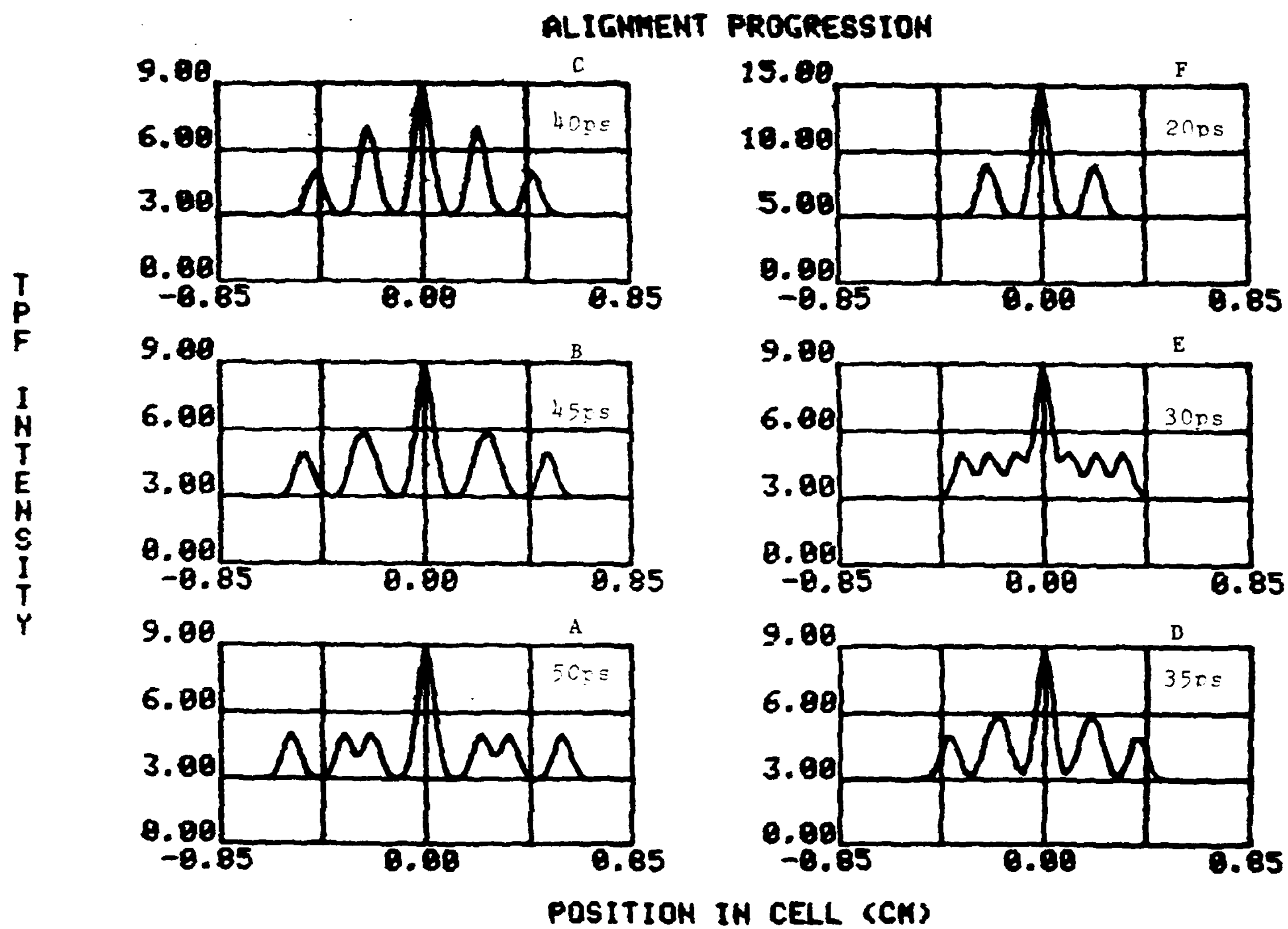
5. Decrease the next optical delay d_i until a second satellite is observed to overlap with the first satellite pulse and the ratio R becomes $i/(i - 1)$.

6. Repeat steps 4 and 5 until all delays have been adjusted.

7. Record the micrometer reading, D_i for each optical delay and calculate its zero position, O_i with respect to d_1. The O^{th} position of the i^{th} optical delay is the micrometer setting that will cause the i^{th} pulse to overlap with the first pulse,

$$O_i = \Delta_i C/2 - D_i$$

8. All subsequent alignments can then be made using the micrometer scale readings (with the appropriate offset O_i) and equation 15.



ALIGNMENT PROGRESSION
C
40ps
F
20ps
B
45ps
E
30ps
A
50ps
D
35ps
TPF INTENSITY
POSITION IN CELL (CM)

FIGURE 17

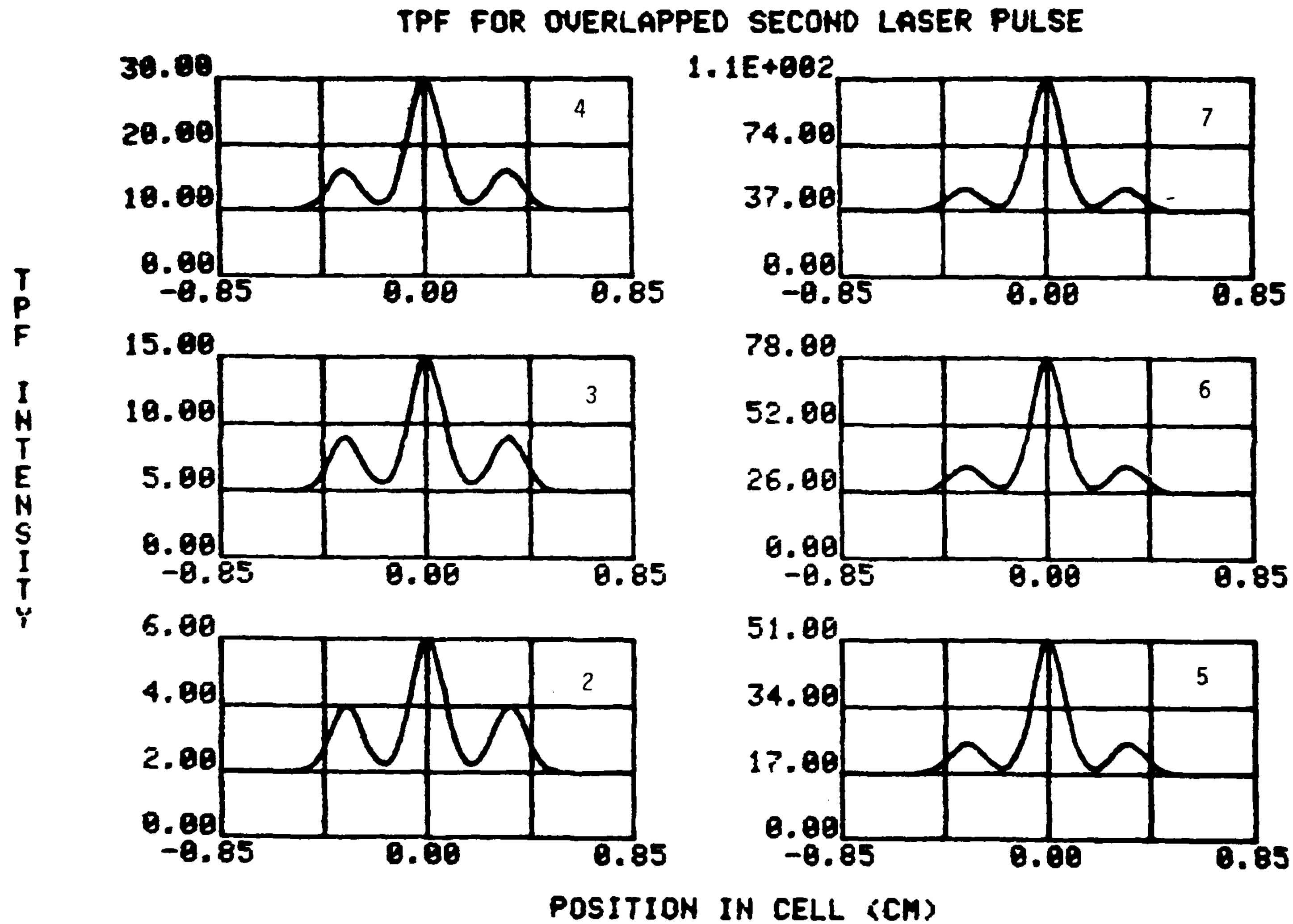

TPF FOR OVERLAPPED SECOND LASER PULSE
TPF INTENSITY
30.00
20.00
10.00
0.00
-0.85
0.00
0.85
4
1.1E+002
74.00
37.00
7
15.00
10.00
5.00
3
78.00
52.00
26.00
6
6.00
4.00
2.00
2
51.00
34.00
17.00
5
POSITION IN CELL (CM)

FIGURE 18

A typical progression of images observed in the TPF cell is shown in figure 17. The third optical delay is to be adjusted as described above, $T_i = 10$ ps, and $\Delta_2 = 20$ ps has already been set. Δ_3 is to be set to $\Delta_3 = 2 \times \Delta_2$. In A, Δ_3 has been adjusted to 50 ps. It can be observed that seven pulses now occur with the TPF cell. When the alignment procedure is completed, a maximum of 5 pulses will be observed. In B the alignment has been improved in this case $\Delta_3 = 45$ ps, the correct number of pulses are observed but the ratio R is not maximized. In C, $\Delta_3 = 40$ ps the correct value, here the ratio R is $i/(i - 1) = 3/2$. If Δ_3 is continued to be decreased, other TPF spectra are observed. In D, $\Delta_3 = 35$ psec and in E, $\Delta_3 = 30$ ps. Finally in F, $\Delta_3 = \Delta_2 = 20$ ps such that the 2nd and 3rd pulses overlap. The ratio R for this case is given by $(1 + (n - 1)^2)/(n - 1)$. It is also possible to define an alignment procedure using this position. In this case the objective would be to set $\Delta_n = \Delta_{n-1} = \ldots = \Delta_3 = \Delta_2$. A progression of these spectra for n = 2 thru 7 is shown in figure 18.

PEAK TO BACKGROUND RATIO

An extensive study was made of the dependence of the two-photon fluorescence contrast ratio on laser pulse intensity for attenuating media. These results were published[5] in the Journal of Applied Physics. A reprint of this article is provided in Appendix 5. (In the reprint, α should be replaced by $-\alpha$ in equation 2 and equations 3a and 3b should be credited to reference 6). In addition, the peak to background ratio, P/B, was investigated for dependence on β when the intensity of the laser pulse is held constant. Results for a laser peak intensity of 40 GW/cm^2 is presented in figure 19.

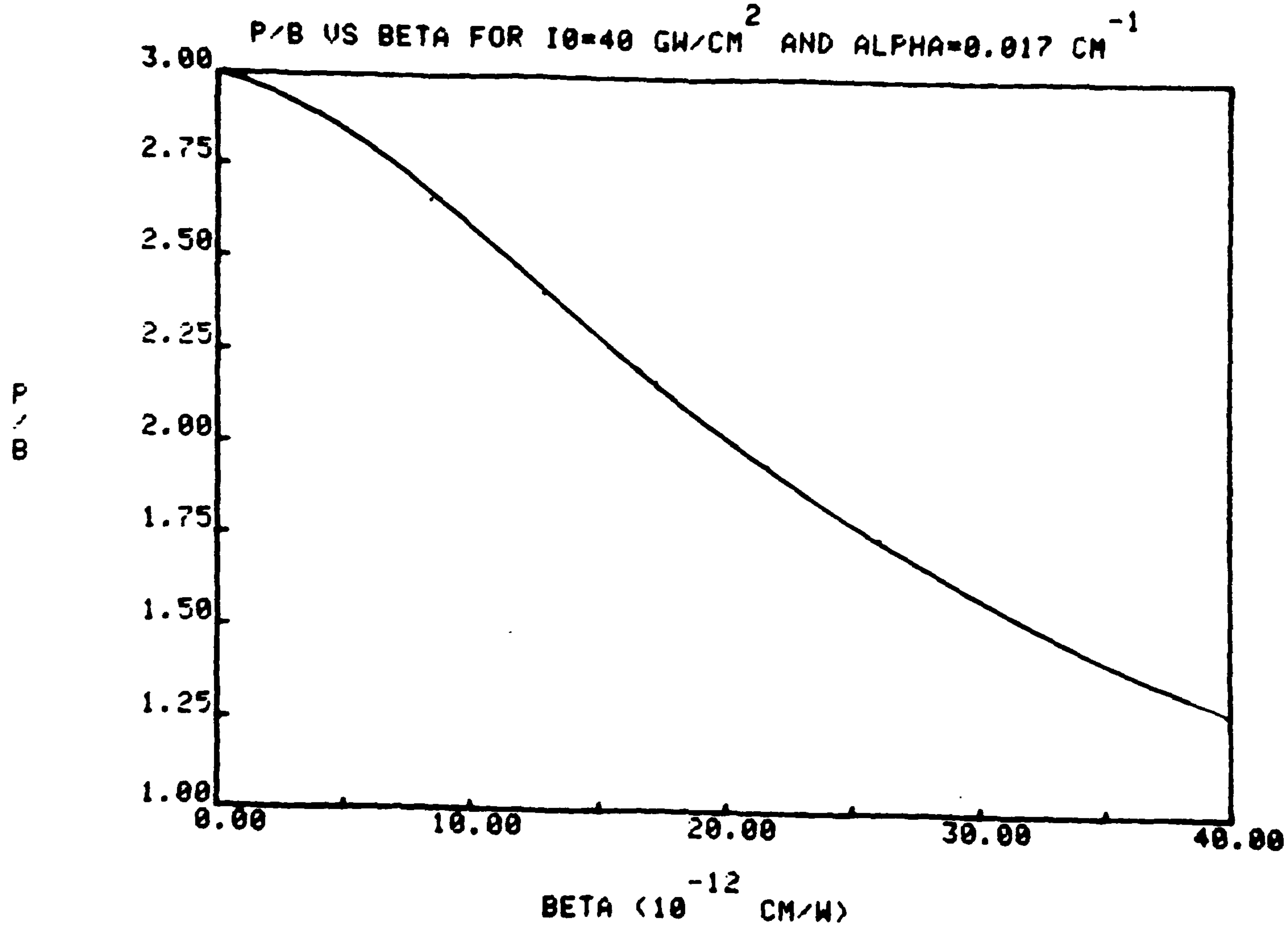

P/B VS BETA FOR I0=40 GW/CM2 AND ALPHA=0.017 CM-1
P/B
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.00
10.00
20.00
30.00
40.00
BETA (10-12 CM/W)

FIGURE 19

FWHM VS β FOR SINGLE PULSES

If two-photon fluorescence is considered for a single laser pulse when $\beta \neq 0$, the FWHM for the laser pulse, ε, will not equal the FWHM of the TPF spectrum, ΔT. A study was conducted to determine variations in ΔT as a function of laser pulse peak intensity. The results are presented graphically in figure 20. For the case presented, $\beta = 1.28 \times 10^{-11}$ cm/W, $\alpha = 0.017$ cm^{-1} and $\varepsilon = 8$ ps. In figure 21, the same data are presented with the corresponding peak to background ratio, P/B, as the independent variable. P/B is defined as the ratio of the TPF intensity at Z = 0 to the TPF intensity at the edge of the cell.

In both graphs, the FWHM values represent direct measurements from the TPF spectrum. An algorithm was developed to rapidly determine ΔT from a digitized TPF spectrum. The algorithm requires operator intervention to define peak bounds and the peak maximum. Background points are fitted with a second degree polynominal function in which the background point at Z = 0 is forced to equal 1/3 the peak maximum at Z = 0. (Theory predicts that the peak to background ratio is 3:1 when both peak and background intensities are measured at Z = 0). Fitted background points are subtracted from the TPF spectrum and the resulting points in the vicinity of peak are fitted to a Gaussian function by least squares techniques. A listing of the algorithm is given in Appendix 6.

FLUORESCENT DECAY

If a single laser pulse passes through a TPF cell in which α and β are not zero, information about the laser pulse can be determined from the fluorescent decay spectrum.

The fluorescent intensity of the TPF medium is given by the relationship:

$$F(Z) = \int_{-\infty}^{\infty} I^2(t - z/V)\, dt \qquad (16)$$

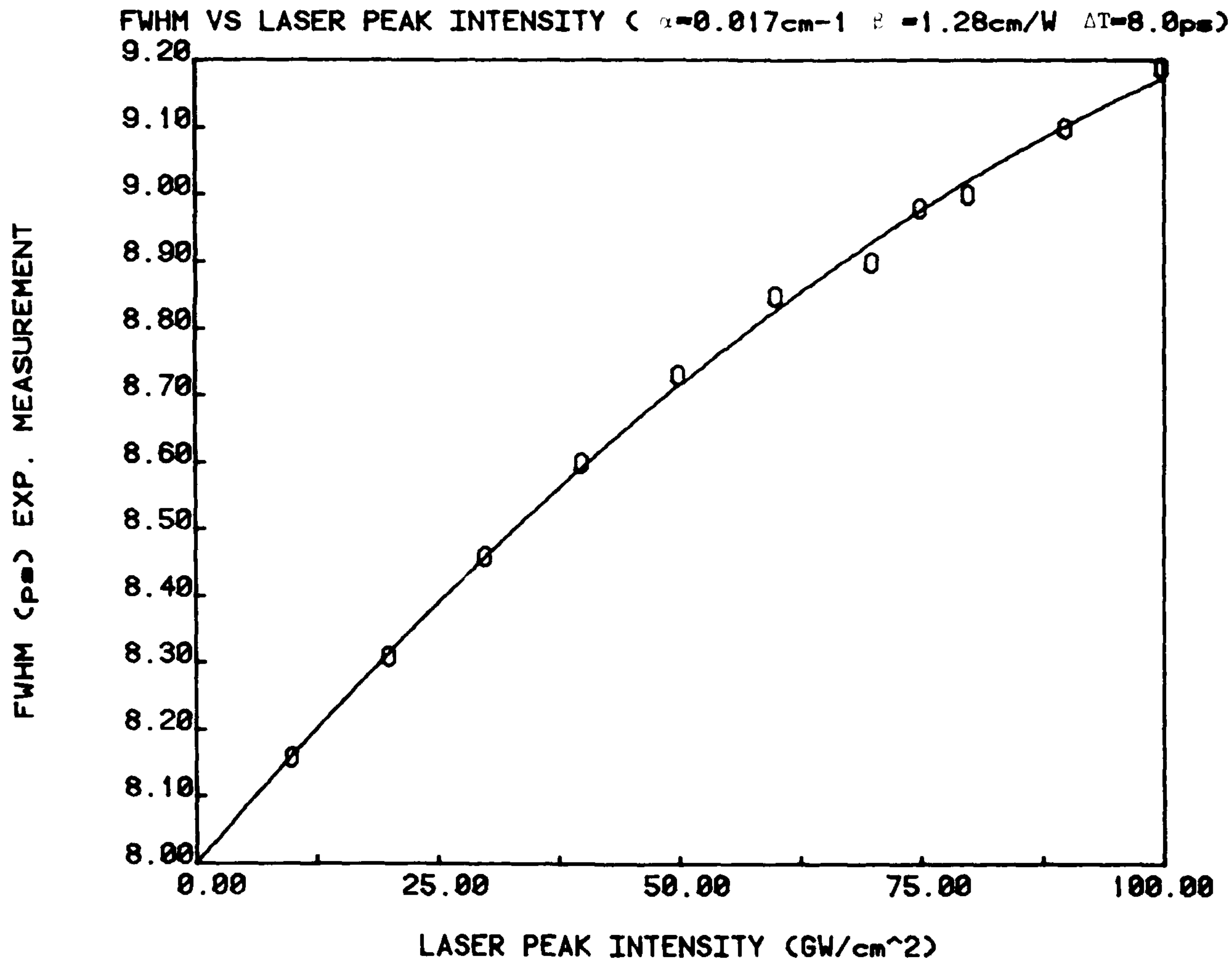

FWHM VS LASER PEAK INTENSITY (α=0.017cm-1 β =1.28cm/W ΔT=8.0ps)
9.20
9.10
9.00
8.90
8.80
8.70
8.60
8.50
8.40
8.30
8.20
8.10
8.00
0.00
25.00
50.00
75.00
100.00
FWHM (ps) EXP. MEASUREMENT
LASER PEAK INTENSITY (GW/cm^2)

FIGURE 20

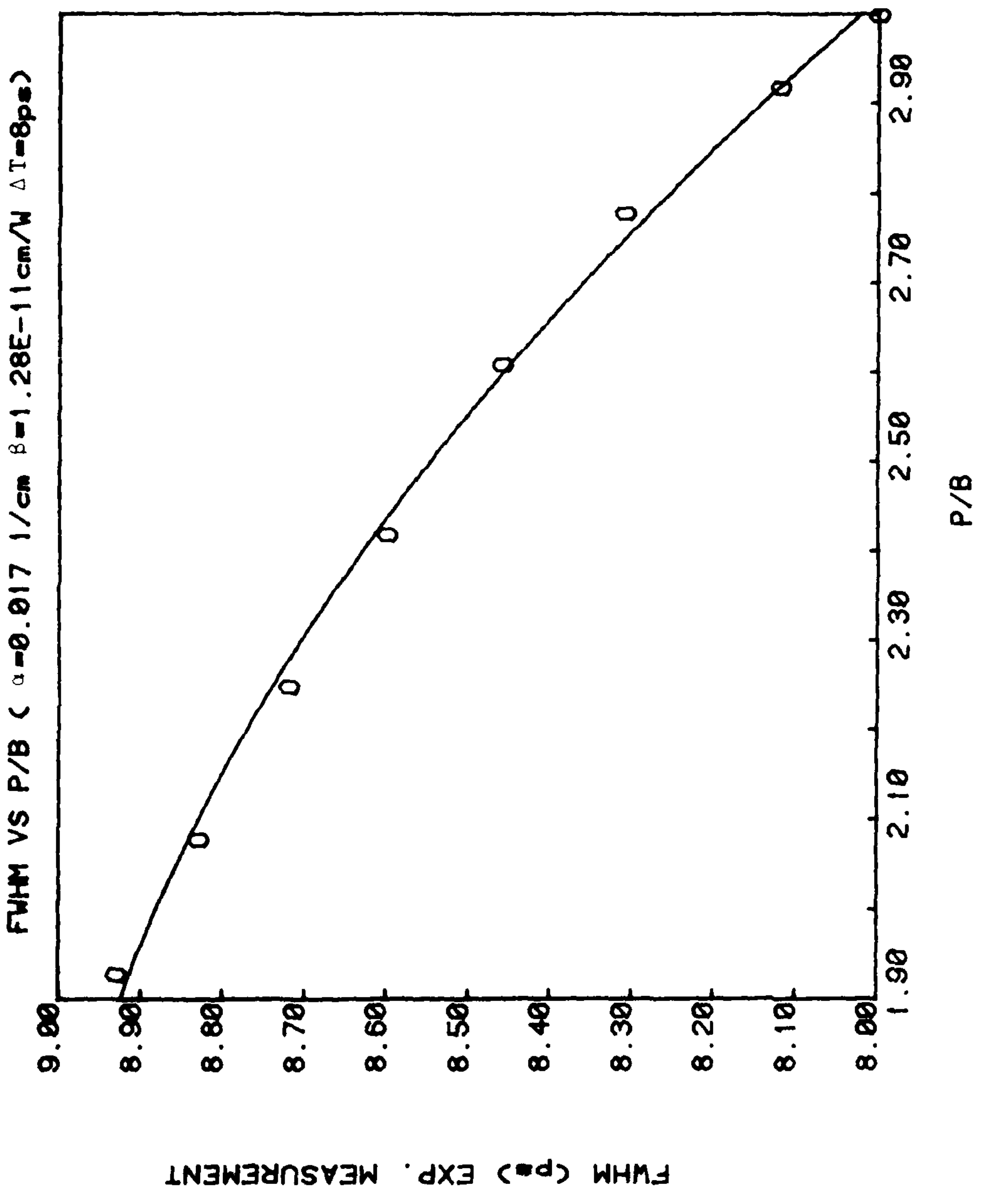

FWHM VS P/B (α=0.017 1/cm β=1.28E-11cm/W ΔT=8ps)
9.00
8.90
8.80
8.70
8.60
8.50
8.40
8.30
8.20
8.10
8.00
1.90
2.10
2.30
2.50
2.70
2.90
P/B
FWHM (ps) EXP. MEASUREMENT

FIGURE 21

where $I(t - z/v) = I_1(t - z/v)$ in equation 2. For analysis, F(z) is normalized so that the intensity at the edge of the cell is 1.

$$f(z) = F(z)/F(d). \tag{17}$$

Equation 17 has been evaluated for several values of α, β and I_o. For all cases, a cell length of 1.7 cm was used. In the accompanying figures, the abscissa ranges from 0 to 1.7 cm, thus the center of the cell is located at Z = 0.85 cm.

In figure 22, four graphs are presented for various values of α and β. In these graphs the upper curve corresponds to a laser peak intensity of 5 GW/cm^2. Successively lower curves correspond to peak intensity of 10, 20, 30, 40, and 60 GW/cm^2. Careful examination of the curves indicates that one of the parameters α, β or I_o could be determined without difficulty. But care must be taken in determining any two of these parameters. Since α can be easily obtained by other methods, the technique can be applied to determine β or I_o. In figure 23, f(z) at the center of the cell, $f(Z_o)$, is plotted against β. For this graph, I_o = 40 GW/cm^2 and α = 0.017 cm^{-1}. In figure 24, $f(Z_o)$ is plotted against laser peak intensity. Here α = 0.017 cm^{-1} and $\beta = 1.28 \times 10^{-11}$ cm/W. This final graph shows the usefulness of using this procedure to determine laser peak intensity. All curves in figures 22 to 24 are independent of the laser pulse width.

Finally, the P/B ratio that would be measured for these laser pulses in a TPF setup can be deduced from the fluorescent decay curve with the relationship

$$\frac{P}{B} = \frac{6f(Z_o)}{1 + f(Z_m)} \tag{18}$$

where $f(Z_m)$ is the fluorescent intensity at the point where the pulse exits from the cell.

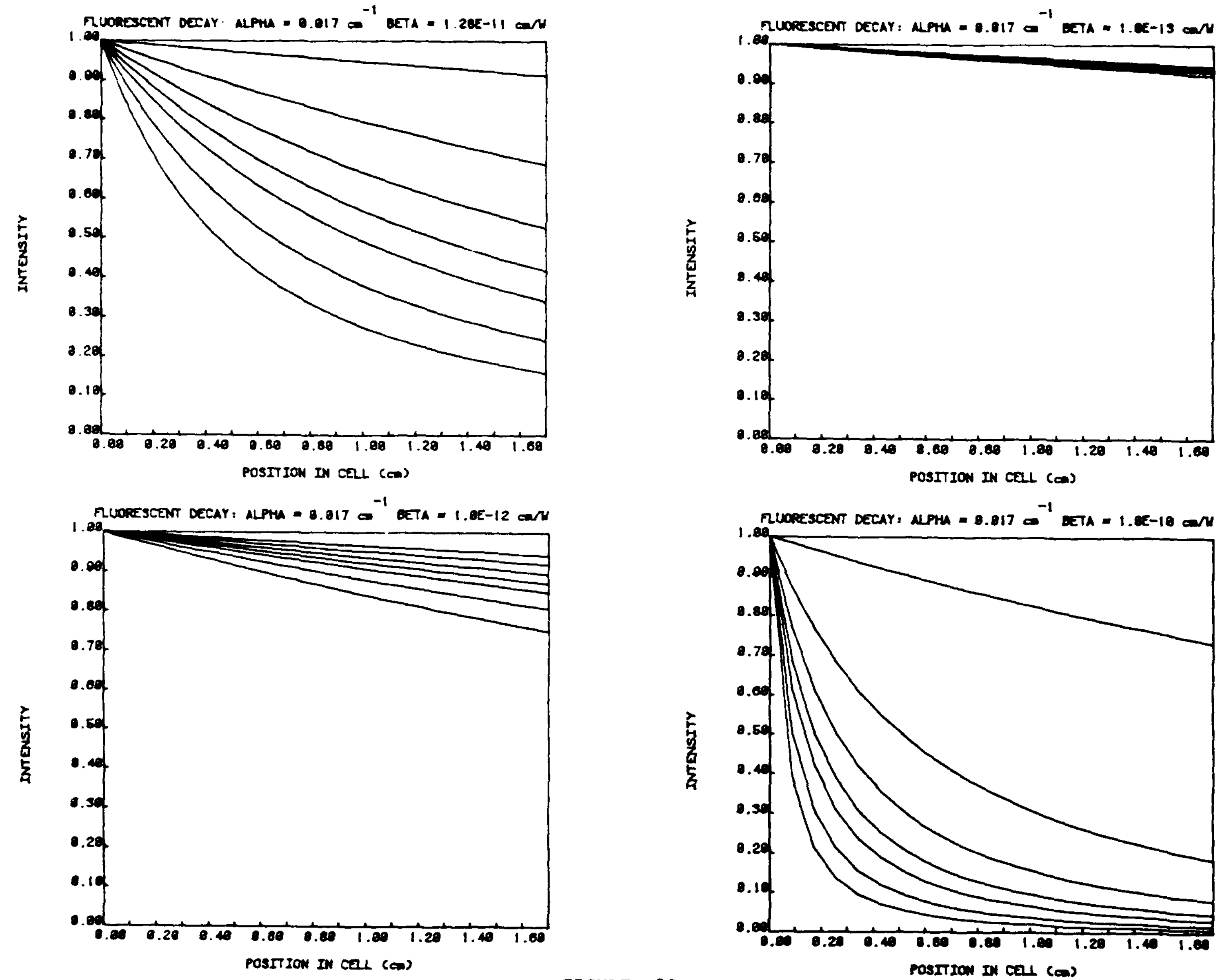

FLUORESCENT DECAY: ALPHA = 0.017 cm^{-1} BETA = 1.20E-11 cm/W
FLUORESCENT DECAY: ALPHA = 0.017 cm^{-1} BETA = 1.0E-13 cm/W
FLUORESCENT DECAY: ALPHA = 0.017 cm^{-1} BETA = 1.0E-12 cm/W
FLUORESCENT DECAY: ALPHA = 0.017 cm^{-1} BETA = 1.0E-10 cm/W
INTENSITY
POSITION IN CELL (cm)
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.20
1.40
1.60

FIGURE 22

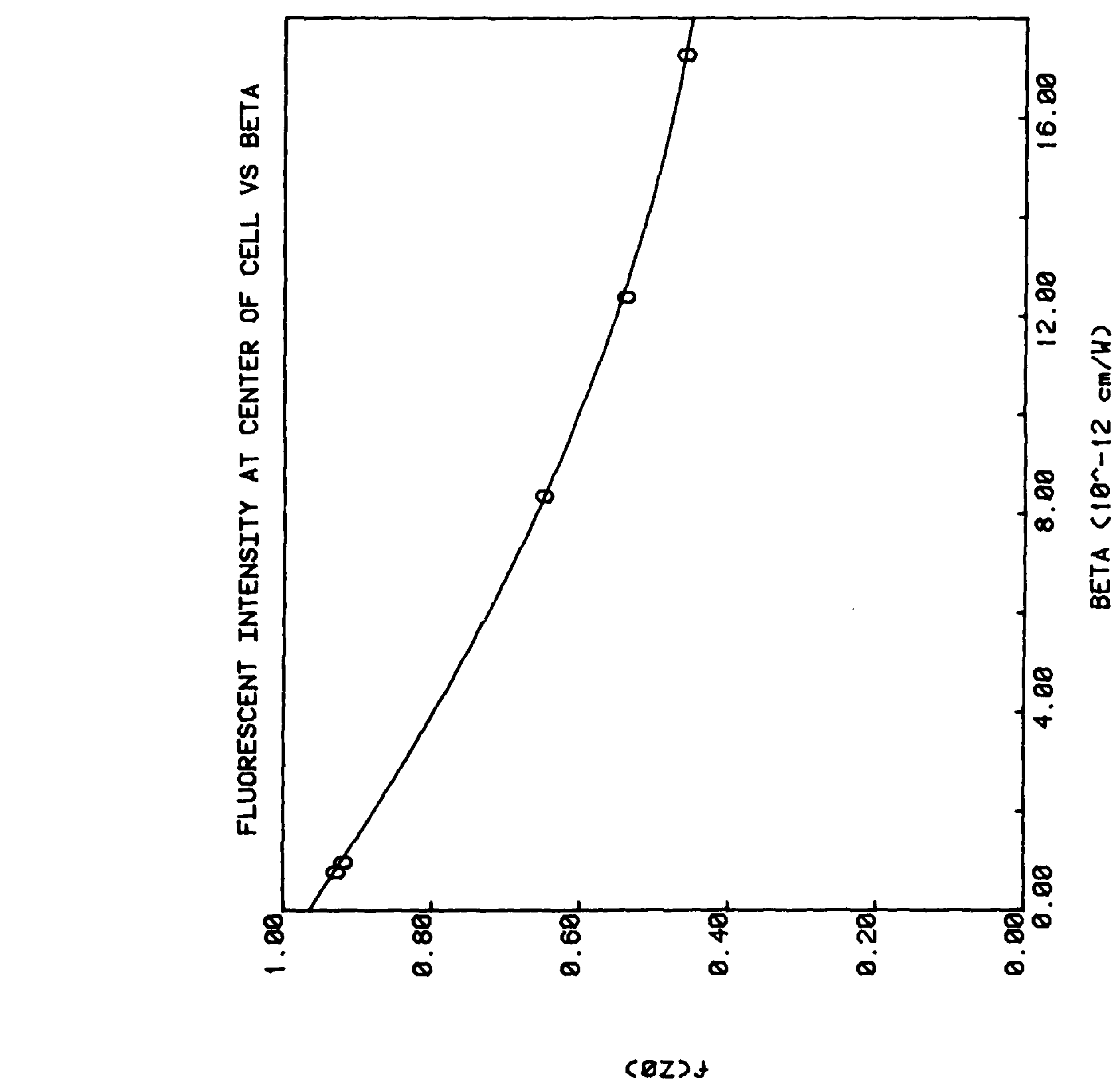

FLUORESCENT INTENSITY AT CENTER OF CELL VS BETA
f(Z0)
1.00
0.80
0.60
0.40
0.20
0.00
0.00
4.00
8.00
12.00
16.00
BETA (10^-12 cm/W)

FIGURE 23

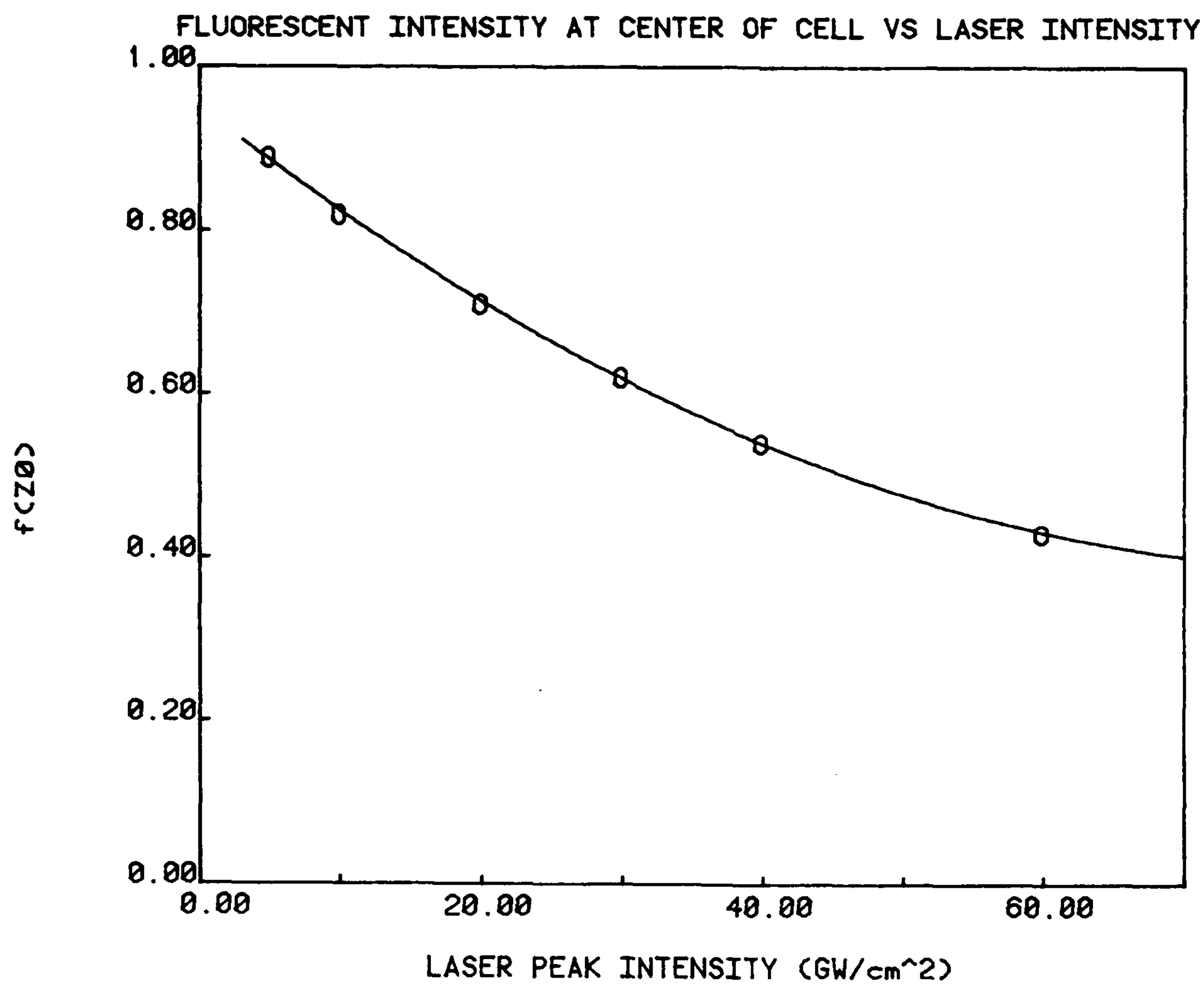

FLUORESCENT INTENSITY AT CENTER OF CELL VS LASER INTENSITY
f(Z0)
1.00
0.80
0.60
0.40
0.20
0.00
0.00
20.00
40.00
60.00
LASER PEAK INTENSITY (GW/cm^2)

FIGURE 24

APPENDIX 1: Intensity Function when $\alpha = 0$.

If $\alpha = 0$ and $\beta \neq 0$, then consider

$$\lim_{\alpha \to 0} \frac{\beta}{\alpha} \left[1 - \mathrm{EXP}(-\alpha(d \pm Z)) \right]$$

$$= (\beta/\alpha) \left[1 - (1 - \alpha(d \pm Z)) \right]$$

$$= (\beta/\alpha)\, \alpha\, (d \pm Z)$$

$$= \beta(d \pm Z).$$

The denominators in equations 2 and 3 should be adjusted accordingly.

APPENDIX 2: Number of TPF Peaks

From equation 4, C_{ij} has a maximum of $N = N' + N_o$, where

$$N' = 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (1) = 2 \sum_{i=1}^{n-1} (n-i) = 2n(n-1) - n(n-1)$$

$$= n^2 - n.$$

In addition, one peak occurs at $Z = 0$ thus $N_o = 1$, therefore

$$N = n^2 - n + 1.$$

APPENDIX 3: Height of Satellite and Main TPF Pulses

Consider equation 4 when $n = 2$, $K = 1$, and $\alpha = 0$. Let B be terms in $S_2(Z)$ that contribute to background signal only, then

$$B = (\pi/2)^{\frac{1}{2}} \left\{ T_1 I_{01}^2 + T_2 I_{02}^2 + \frac{2^{\frac{1}{2}} T_1 T_2 I_{01} I_{02}}{(T_1^2 + T_2^2)^{\frac{1}{2}}} \, \mathrm{EXP}\left(- \frac{\Delta_2^2}{T_1^2 + T_2^2} \right) \right\}$$

If in addition, the laser pulses are isolated so $\Delta_2^2 >> T_1^2 + T_2^2$, the ratio of the height of the main pulse above the background to the height of the satellite pulse above the background is R, where

$$R = \frac{(1 + \sigma\theta)\sqrt{1 + \sigma}}{\sqrt{2}\,\theta\sigma} = \frac{S(0) - B}{S(V\Delta_2/2) - B},$$

where $\theta = I_{02} / I_{01}$ and $\sigma = T_2 / T_1$. R has a minimum value of 2 at $\theta = \sigma = 1$

APPENDIX 4: Peak to Background Ratio when $\beta = 0$

R' is obtained from equation 4 when n=1, then

$$R' = \frac{S_1(0)}{S_1(-d)} = \frac{(\pi/2)^{\frac{1}{2}}\,(EXP(-2\alpha d) + K^2EXP(-2\alpha d))\;T_1 I_{01}^2}{(\pi/2)^{\frac{1}{2}}\,(1 + K^2EXP(-4\alpha d))\;T_1 I_{01}^2}$$

$$+ \frac{4(\pi/2)^{\frac{1}{2}}\,EXP(-2\alpha d)\;T_1 I_{01}^2\,K}{(\pi/2)^{\frac{1}{2}}\,(1 + K^2EXP(-4\alpha d))\;T_1 I_{01}^2}$$

$$= \frac{EXP(-2\alpha d)(1 + K^2 + 4K)}{1 + K^2EXP(-4\alpha d)}$$

APPENDIX 5: Reprint of Reference Five

Dependence of the decrease in contrast ratios on the intensity of the laser pulse for two-photon fluorescence[a)]

D. D. Venable
Department of Physics, Hampton Institute, Hampton, Virginia 23668

J. Taboada
USAF School of Aerospace Medicine, Laser Effects Branch (RZL), Brooks AFB, Texas 78235

(Received 2 April 1979; accepted for publication 24 May 1979)

The contrast ratio for two-photon fluorescence (TPF) is found to depend on the laser pulse intensity. The peak-to-background ratio is observed to decrease from the ideal value of 3 to about 2.1 as the intensity of the single Nd-glass laser ultrashort pulse is increased from 15 to 60 GW/cm^2. Parametric analysis of the autocorrelation function for the pulse amplitude shows that this effect can be explained by two-photon attenuation of the laser pulse by the TPF medium.

PACS numbers: 42.60.He, 42.60.Kg, 42.65. – k

The two-photon fluorescence (TPF) technique of Giordmaine *et al.*[1] is a well-known procedure for ultrashort laser pulse time duration measurements.[2] Care must be taken in interpreting the fluorescence traces observed in these measurements.[1-4] Ideally, mode-locked pulses should exhibit a peak-to-background ratio (P/B) of 3. P/B values ranging from about 2.3 to 3 have been reported in the literature.[5] For conditions employed by many researchers, P/B may depend on the attenuating characteristics of the TPF medium.[6] We recently reported on a low-light-level electro-optic TPF system with sufficient dynamic range to experimentally observe a decrease in P/B as laser pulse intensity increases.[7] A simple model of the TPF experimental arrangement described in Ref. 7 is shown in Fig. 1. The time-integrated TPF intensity, $S(z)$, is proportional to the autocorrelation of the laser pulse intensity as follows:

$$S(z) \propto \int_{-\infty}^{\infty} I_1^2(t - z/V)\,dt + \int_{-\infty}^{\infty} I_2^2(t + z/V)\,dt + 4\int_{-\infty}^{\infty} I_1(t - z/V) I_2(t + z/V)\,dt. \qquad (1)$$

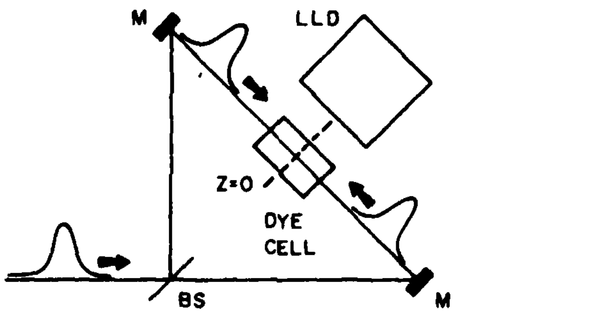


FIG. 1. Typical TPF arrangement. BS—beam splitter; M—total reflecting mirrors; LLD—low-light-level electro-optics detecting system. (Details of the system are given in Ref. 7.)

The position of maximum overlap in the cell is given by $z = 0$ and V is the group velocity of the laser pulse in the TPF medium. I_1 and I_2 are the intensities of the laser pulses in each arm of the triangular arrangement in Fig. 1. The absorption of laser light when traveling in a medium with appreciable one- and two-photon absorption cross sections is given by

$$\frac{\partial I(t - z/V)}{\partial z} = \alpha I(t - z/V) - \beta I^2(t - z/V). \qquad (2)$$

Here α and β are respectively the one- and two-photon attenuation coefficients of the TPF medium. For temporally Gaussian but spatially uniform pulses, with $1/e$ half-width, τ, I_1 and I_2 are given by

$$\begin{aligned} I_1(t - z/V) &= I_0 \exp\{-[(t - z/V)^2\tau^{-2}]\} \exp[-\alpha(d + z)] \\ &\quad \times (1 + (\beta/\alpha) I_0 \exp\{-[(t - z/V)^2\tau^{-2}]\} \\ &\quad \times \{1 - \exp[-\alpha(d + z)]\})^{-1} \end{aligned} \qquad (3a)$$

and

$$\begin{aligned} I_2(t + z/V) &= I_0 \exp\{-[(t + z/V)^2\tau^{-2}]\} \exp[-\alpha(d - z)] \\ &\quad \times (1 + (\beta/\alpha) I_0 \exp\{-[(t + z/V)^2\tau^{-2}]\} \\ &\quad \times \{1 - \exp[-\alpha(d - z)]\})^{-1}. \end{aligned} \qquad (3b)$$

The dye cell path length is $2d$ and I_0 is the peak intensity of each of the pulses. In general, no simple solution exists for Eq. (1) for these values of I_1 and I_2.

In our experiment, single pulses were extracted from the early portion of a train of mode-locked Nd-glass laser pulses. Details of the laser system are reported elsewhere.[8] The full width at half-maximum of the laser pulses was determined to be 8 ± 1 ps. The energy of each pulse was monitored with a UDT Model 51A Laser Energy Evaluator that was cross-calibrated with a Hadron Cone Calorimeter. The TPF medium consisted of $\simeq 5 \times 10$ 'M solution of Rhodamine 6G in ethanol and the cell itself was 1.70 cm long. The single-photon attenuation coefficient, α, was determined to

[a)]Supported in part by the Air Force Office of Scientific Research, Air Force Systems Command, USAF, under Grant No. AFOSR-79-0036.

be 0.017 cm^{-1} with a Beckman spectrophotometer using low-intensity 1.06-μm radiation.

TPF contrast ratios obtained for intensities from $\simeq 15$ to $\simeq 60$ GW/cm^2 are given in Fig. 2. For the lower intensities, P/B is seen to equal the ideal value of 3 within experimental error. But as the laser intensity increases, P/B decreases significantly. Typical TPF oscillograms are given in Ref. 7. Theoretical values for P/B were obtained by numerical integration of Eq. (1). To best estimate the experimental measurements, P/B was defined to be the ratio of the fluorescence intensity at the central maximum to that at the edge of the cell. Thus,

$$P/B = S(0)/S(\pm d). \tag{4}$$

Using data-fitting techniques, we determined that a value of 1.28×10^{-11} cm/W for β would provide the best fit of Eq. (4) to the experimental data. For multiphoton processes, the n-photon absorption cross section, σ_n, is related to the n-photon attenuation coefficient, δ_n, by the relationship

$$\sigma_n = \frac{\delta_n \hbar\omega}{n(n-1)N}, \tag{5}$$

where N is the number of n-photon absorbing molecules per unit volume and $\hbar\omega$ is the laser photon energy. For our experiment, where $n = 2$ and $\beta = \delta_2$, we obtain a value of $\sigma_2 = 3.93 \times 10^{-49}$ cm^4 s. This value is somewhat larger than those reported in the literature,[9] but not unreasonable when one considers the large disparity in the reported values.

For sufficiently low intensities $(\beta/\alpha)I_0 \ll 1$. For this situation, an analytical solution exists for Eq. (1) and the P/B obtained for Eq. (4) is a maximum. This maximum value can be approximated by the expression

$$(P/B)_{max} \simeq \frac{3}{\cosh(2\alpha d)}. \tag{6}$$

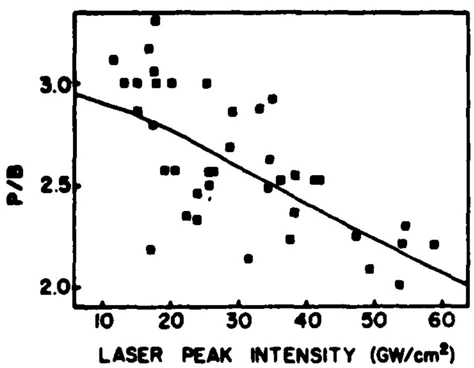


FIG. 2. Experimental (■) and calculated (solid line) TPF contrast ratios for $\simeq 5 \times 10^{-5}$ *M* solution of Rhodamine 6G in ethanol. The calculated ratios are defined to be $S(0)/S(\pm d)$.

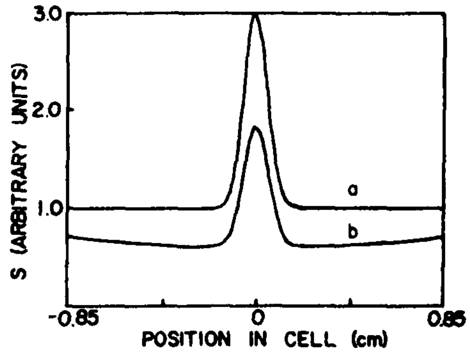


FIG. 3. Calculated TPF oscillograms for (a) $\alpha = 0$ and $\beta = 0$, and (b) $\alpha = 0.017$ cm^{-1} and $\beta = 1.28 \times 10^{-11}$ cm/W. Peak intensity of each pulse is 30 GW/cm^2 and $2d = 1.70$ cm. $S(z)$ is normalized so that the maximum fluorescence signal for case (a) has a value of 3.

Our experimental conditions yield $(P/B)_{max} \simeq 3.00$. Thus, no significant decrease in P/B will be observed at low intensities. Comparison of the theoretical and experimental results thus indicates that the observed variations in P/B are attributed to the finite value of β, i.e., two-photon attenuation of the laser pulse by the TPF medium.

Finally, numerical results for Eq. (1) are given in Fig. 3. We note that for our experimental conditions the exponential decay in the TPF signal might not be apparent under superficial examination of the TPF oscillograms. The effects are even more obscured when large data fluctuations occur in the oscillograms or in densitometer scans. These data emphasize the need for researchers to minimize one- and two-photon attenuation of the laser beams in TPF media. This requires (a) low dye concentrations and (b) low laser intensities. These two requirements suggest the desirability of low-light-level TPF detection systems.

[1] J.A. Giordmaine, P.M. Rentzepis, S.L. Shapiro, and K.W. Wecht, Appl. Phys. Lett. 11, 216 (1967).
[2] E.P. Ippen and C.V. Shank, in *Ultrashort Light Pulses: Picosecond Techniques and Applications*, edited by S.L. Shapiro (Springer-Verlag, Berlin, 1977), Vol. 18, p. 83.
[3] H.P. Weber, Phys. Lett. 27, 321 (1968).
[4] D.J. Bradley, T. Morrow, and M.S. Petty, Opt. Commun. 2, 1 (1970).
[5] D.J. Bradley and C.H. New, Proc. IEEE 62, 313 (1974).
[6] J.H. Bechtel and W.L. Smith, J. Appl. Phys. 46, 5055 (1975).
[7] J. Taboada and D.D. Venable, J. Appl. Phys. 49, 5669 (1978).
[8] J. Taboada and R.W. Ebbers, Appl. Opt. 14, 1759 (1975).
[9] J.P. Herman and J. Ducuing, Opt. Commun. 6, 101 (1972).

APPENDIX 6: Program Listings

All programs are written in BASIC for the Tektronix 4051 Graphics Computer with 16K memory.

1. $S_n(Z)$---Numerical Integration

2. $S_n(Z)$--- Closed Form for Beta = 0

3. Laser Pulse F W H M from T P F Spectra (Requires MATRIX ROM)

4. Graphics Algorithm

```
1 REM---NUMERICAL INTEGRATION FOR S(Z)---06/79-03
100 INIT
110 REM N0=# OF PULSES IN TRAIN
120 N0=1
130 GOSUB 650
140 GOSUB 1+(D1=1) OF 740,1910
150 GOSUB 240
160 GOSUB 360
170 GOSUB 540
180 GOSUB -1=D1 OF 690
190 GOSUB 2+D1 OF 820,930,1050
200 GOSUB 1+(D1=1) OF 1090,1110
210 GOSUB Q1*(2+D1) OF 1240,1320
220 GOSUB Q2 OF 1370
230 END
240 DIM I0(N0),P(N0),D(N0)
250 REM INTENSITY=I0(I), 1/e HW=P(I), PULSE SEPAR=D(I)
260 REM DATA FORMAT : I0(GW/CM↑2), TAU(PSEC), DELTA(PSEC),...
270 DATA 1,6,0,1,6,20,1,6,40,1,6,60,1,6,80
280 DATA 1,6,100
290 FOR I=1 TO N0
300 READ I0(I),P(I),D(I)
310 NEXT I
320 I0=I0*1.0E+9
330 P=P*1.0E-12
340 D=D*1.0E-12
350 RETURN
360 REM
370 REM P4=K(EQ. 3),V=VEL. IN MEDIUM,Z7=CELL LENGTH,Z3=STEP SIZE CM.
380 P4=1
390 V=2.22E+10
400 Z7=0.8436
410 Z1=-Z7
420 Z2=Z7
```

```
430 Z3=0.222
440 REM Z5=ALPHA, Z6=BETA
450 Z5=0
460 Z6=0
470 D(1)=0
480 N=INT((Z2-Z1)/Z3+1.1)
490 N3=3*SQR(2)/(6*SQR(PI)*P(1)*I0(1)↑2)
500 REM MOD EXP FUNCTION
510 DEF FNE(X)=(ABS(X)<100)*EXP((ABS(X)<100)*X)
520 D2=0
530 RETURN
540 REM J2=1 PRI CONV1; J3=1 PRI CONV2; J4=0 CK FOR CONV; C=CONV CRIT
550 REM  K=# OF PTS; B=LIMITS; J5=0 BKG ONLY
560 J2=0
570 J3=0
580 J4=0
590 J5=1
600 C=0.0033
610 B=1.0E-10
620 K=6
630 K1=K
640 RETURN
650 PRINT "ENTER 1 FOR SINGLE VALUE "
660 PRINT USING "7TFA5XFA":"0 FOR CORE STORAGE_","-1 FOR TAPE STORAGE"
670 INPUT D1
680 RETURN
690 PRINT "PREPARE TAPE & ENTER FILE # FOR DATA STORAGE ";
700 INPUT D2
710 GOSUB 1900
720 PAGE
730 RETURN
740 PRINT " ENTER 'YES' TO PRINT INTENSITY SPECTRUM G";
750 GOSUB 1850
760 Q1=Q
770 PRINT " ENTER 'YES' TO PLOT SPECTRUM G";
```

```
780 GOSUB 1800
790 Q2=Q
800 PAGE
810 RETURN
820 FIND D2
830 WRITE N
840 FOR Z=Z1 TO Z2 STEP Z3
850 GOSUB 1700
860 GOSUB 1960
870 B=B/(2-(J4=1))
880 K=K1
890 WRITE Z,S
900 NEXT Z
910 CLOSE
920 RETURN
930 L=0
940 DIM X(N),Y(N)
950 FOR Z=Z1 TO Z2 STEP Z3
960 L=L+1
970 GOSUB 1700
980 GOSUB 1960
990 X(L)=Z
1000 Y(L)=S
1010 B=B/(2-(J4=1))
1020 K=K1
1030 NEXT Z
1040 RETURN
1050 GOSUB 1700
1060 GOSUB 1960
1070 PRINT USING "18A-2D.3DFA2D.4D":"LZ =  ",Z,"WIS(Z) =I ",S
1080 RETURN
1090 PRI USI 1100:"LZ-INIT =";Z1;"CM_Z-LAST =";Z2;"CM_Z-STEP =";Z3;"CM"
1100 IMAGE 18A-2D.3D1X20A-2D.3D1X20A-2D.3D1X2A
1110 PRINT "VELOCITY =I ";V;" CM/SEC_NORM =I ";N3;"_ATTEN =I ";P4
1120 PRINT "BETA =I ";Z6;" CM/W"
```

```
1130 PRINT "ALPHA =I ";Z5;" 1/CM_CELL LENGTH =I ";Z7*2;" CM"
1140 PRINT "FILED ON:I ";D2;"_RESOLUTION:I ";Z3/V;" SEC"
1150 IF J5=1 THEN 1170
1160 PRINT "BKG--CALC."
1170 PRI "_ #  I0(GW/CM↑2)"," 1/e-HW(PSEC)"," FWHM(PSEC)"," SEP.(PSEC)"
1180 IMAGE 2D,4T,4D.2D,19T,4D.2D,39T,4D.2D,58T,4D.2D
1190 J=1.0E+12
1200 FOR I=1 TO N0
1210 PRINT USING 1180:I,I0(I)*1.0E-9,P(I)*J,P(I)*2.354/SQR(2)*J,D(I)*J
1220 NEXT I
1230 RETURN
1240 PRINT "JZ"," S(Z)"
1250 FIND D2
1260 READ @33:N
1270 FOR I=1 TO N
1280 READ @33:Z,S
1290 PRINT USING "-2D.3D10X2D.3D":Z,S
1300 NEXT I
1310 RETURN
1320 PRINT "JZ"," S(Z)"
1330 FOR L=1 TO N
1340 PRINT USING "-2D.3D10X2D.3D":X(L),Y(L)
1350 NEXT L
1360 RETURN
1370 REM
1390 DELETE G,G1,G2
1400 DIM G(17)
1410 DATA 0,0,0,0,0,0,0,0,1,0,20,120,10,90,33,32,1
1420 RESTORE 1410
1430 READ G
1440 G(1)=N
1450 G(2)=Z1
1460 G(3)=Z2
1470 G(4)=Z7
1480 G(6)=3*N0
```

```
1490 G(7)=N0
1500 G(10)=D2
1510 X$="POSITION IN CELL (cm)"
1520 Y$="INTENSITY"
1530 PRINT "ENTER TITLE: "
1540 INPUT T$
1550 GOSUB 2430
1560 RETURN
1570 F1=0
1580 F2=0
1590 FOR J=1 TO N0
1600 F3=I0(J)*FNE(-(((T-D(J)-Z/V)/P(J))↑2))
1610 F4=I0(J)*FNE(-(((T-D(J)+Z/V)/P(J))↑2))*P4
1620 IF Z5=0 AND Z6=0 THEN 1650
1630 F3=F3*A1/(1+F3*A3)
1640 F4=F4*A2/(1+F4*A4)
1650 F1=F1+F3
1660 F2=F2+F4
1670 NEXT J
1680 F=N3*(F1↑2+F2↑2+4*J5*F1*F2)
1690 RETURN
1700 IF Z5=0 AND Z6=0 THEN 1790
1710 A1=FNE(-Z5*(Z7+Z))
1720 A2=FNE(-Z5*(Z7-Z))
1730 IF Z5<>0 THEN 1770
1740 A3=Z6*(Z7+Z)
1750 A4=Z6*(Z7-Z)
1760 GO TO 1790
1770 A3=(1-A1)*Z6/Z5
1780 A4=(1-A2)*Z6/Z5
1790 RETURN
1800 REM YES RESPONSE
1810 INPUT Q$
1820 Q=POS(Q$,"Y",1)
1830 Q=Q<>0
```

```
1840 RETURN
1850 REM HIGHLITE
1860 FOR I=1 TO 25
1870 PRINT " WRITING ON FILE # ";D2;"GK"
1880 NEXT I
1890 PRINT "J";
1900 RETURN
1910 PRINT " ENTER Z VALUES ";
1920 INPUT Z
1930 Q1=0
1940 Q2=0
1950 RETURN
1960 REM *****SIMPSON'S INTEGRATION FROM - TO + INFINITY*****
1970 IF J4=1 THEN 2020
1980 T=B
1990 GOSUB 1570
2000 S4=F
2010 S2=F
2020 H=2*B/2↑K
2030 T=-B
2040 GOSUB 1570
2050 S=F
2060 T=B
2070 GOSUB 1570
2080 S=(S+F)/3
2090 I2=1
2100 FOR I=-B+H TO B-H STEP H
2110 IF I2=0 THEN 2140
2120 I1=1.33333333
2130 GO TO 2150
2140 I1=0.666666667
2150 T=I
2160 GOSUB 1570
2170 S=S+F*I1
2180 I2=NOT(I2)
```

```
2190 NEXT I
2200 S=S*H
2210 IF J4=1 THEN 2390
2220 S1=(S-S2)/S
2230 IF J2=0 THEN 2250
2240 PRINT "CONV1=";S1;" INT=";S;" B=";B;" K=";K
2250 IF ABS(S1)<=C THEN 2290
2260 S2=S
2270 K=K+1
2280 GO TO 2020
2290 S1=(S-S4)/S
2300 PRINT "GG";
2310 IF J2=0 THEN 2330
2320 PRINT "***CONV2=";S1;" INT=";S;" B=";B;" K=";K
2330 IF ABS(S1)<=C THEN 2380
2340 S4=S
2350 S2=S
2360 B=B*2
2370 GO TO 2020
2380 REM
2390 IF J3=0 THEN 2420
2400 PRI "INTEGRAL =";S;" FOR Z =";Z;"_##CONVERGENCE AT ";C;" FOR B =";
2410 PRINT B/(2-(J4=1));" AND K = ";K-1+(J4=1)
2420 RETURN
2430 REM PLOT ROUTINE
2440 REM
2450 REM
2460 REM GRAPHICS PLOT ALGORITHM APPENDED
2470 REM
2480 REM
2490 RETURN
```

```
1 REM---S(Z) FOR N0 PULSES CLOSED FORM---#2/79-03
100 INIT
110 DELETE C,P1,I0,P,D,X,Y
120 REM N0=# OF PULSES IN LASER TRAIN
130 N0=1
140 REM CONT PROGRAM
150 GOSUB 580
160 GOSUB 1+(D1=1) OF 630,1950
170 GOSUB -1=D1 OF 710
180 GOSUB 290
190 GOSUB 410
200 GOSUB 1550
210 GOSUB (D1=-1)+2*(D1=0)+3*(D1=1) OF 750,840,930
220 GOSUB 1+(D1=1) OF 960,980
230 GOSUB 1 OF 1020
240 GOSUB 1130
250 GOSUB Q1*((D1=-1)+2*(D1=0)) OF 1200,1280
260 GOSUB Q2 OF 1330
270 PRINT "GGGG"
280 RETURN
290 DIM I0(N0),P(N0),D(N0),C(N0,N0),S(5),P1(N0,N0)
300 REM INTENSITY=I0(I), 1/e HW=P(I), PULSE SEPAR=D(I)
310 REM DATA FORMAT : I0(GW/CM↑2), TAU(PSEC), DELTA(PSEC), ...
320 DATA 1,6.0077,0,1,6.0077,50,1,6.0077,100,1,6.0077,150,1,6.0077,200
330 DAT 1,6.0077,250,1,6.0077,300,1,6.0077,350,1,6.0077,400,1,6.0077,450
340 FOR I=1 TO N0
350 READ I0(I),P(I),D(I)
360 NEXT I
370 I0=I0*1.0E+9
380 P=P*1.0E-12
390 D=D*1.0E-12
400 RETURN
410 REM
420 REM P4=K(EQ. 3),V=VEL. IN MEDIUM,Z7=CELL LENGTH,Z3=STEP SIZE CM
```

```
430 P4=1
440 V=2.22E+10
450 Z7=0.8436
460 Z1=-Z7
470 Z2=Z7
480 Z3=0.0222
490 REM Z5=ALPHA, Z6=BETA
500 Z4=0
510 Z5=0
520 N3=3*(2/PI)↑0.5/(6*P(1)*I0(1)↑2)
530 N=INT((Z2-Z1)/Z3+1.1)
540 C=0
550 P1=0
560 D(1)=0
570 RETURN
580 PRINT USING "FA":"LENTER 1 FOR SINGLE VALUE"
590 PRINT USING "7TFA/6TFA":"0 FOR CORE STORAGE","-1 FOR TAPE STORAGE"
600 INPUT D1
610 D2=0
620 RETURN
630 PRINT " ENTER 'YES' TO PRINT TPF SPECTRUM: G";
640 GOSUB 1840
650 Q1=Q
660 PRINT " ENTER 'YES' TO PLOT SPECTRUM: G";
670 GOSUB 1840
680 Q2=Q
690 PAGE
700 RETURN
710 PRINT "INIT TAPE FOR DATA: ENTER FILE #  & WRITE UNIT #: ";
720 INPUT D2,U2
730 GOSUB 1890
740 RETURN
750 FIND @U2:D2
760 PAGE
770 WRITE @U2:N
```

```
780 FOR Z=Z1 TO Z2 STEP Z3
790 GOSUB 1680
800 WRITE @U2:Z,S0
810 NEXT Z
820 CLOSE @U2:
830 RETURN
840 L=0
850 DIM X(N),Y(N)
860 FOR Z=Z1 TO Z2 STEP Z3
870 L=L+1
880 GOSUB 1680
890 X(L)=Z
900 Y(L)=S0
910 NEXT Z
920 RETURN
930 GOSUB 1680
940 PRINT USING "FA19T-1D.3D37TFA55T-1D.3D":"Z =",Z,"HIS =",S0
950 RETURN
960 PRINT USING 970:"Z-INIT =",Z1,"CM_Z-LAST =",Z2,"CM_Z-STEP =",Z3,"CM"
970 IMAGE 3(FA10X-FD.FD1X)FA
980 PRINT "VELOCITY =I ";V;" CM/SEC_NORM =I ";N3;"_ATTEN =I ";P4
990 PRINT "ALPHA =I ";Z5;" 1/CM_BETA =I 0_CELL LENGTH =I ";Z7*2;" CM"
1000 PRINT "FILED ON:I ";D2;"_RESOLUTION:I ";Z3/V;" SEC"
1010 RETURN
1020 Z=0
1030 GOSUB 1680
1040 PRINT USING "1(FA19T2D.3D)":"BKG(@Z=0) =",N3*S(1)*S(3)
1050 PRINT USING "1(FA19T2D.3D)":"PEAK(@Z=0) =",S0
1060 Z=-Z7
1070 GOSUB 1680
1080 PRINT USING "FA19T2D.3D":"BKG(@Z=-d) =",S0
1090 Z=Z7
1100 GOSUB 1680
1110 PRINT USING "FA19T2D.3D":"BKG(@Z=d)  =",S0
1120 RETURN
```

```
1130 PRI "_ #  I0(GW/CM↑2)"," 1/e-HW(PSEC)"," FWHM(PSEC)"," SEP.(PSEC)"
1140 IMAGE 2D,4T,4D.2D,19T,4D.2D,39T,4D.2D,58T,4D.2D
1150 J=1.0E+12
1160 FOR I=1 TO N0
1170 PRINT USING 1140:I,I0(I)*1.0E-9,P(I)*J,P(I)*2.354/SQR(2)*J,D(I)*J
1180 NEXT I
1190 RETURN
1200 PRINT "JZ"," S(Z)"
1210 FIND @U2:D2
1220 READ @U2:N
1230 FOR I=1 TO N
1240 READ @U2:Z,S0
1250 PRINT USING "-2D.3D10X-2D.3D":Z,S0
1260 NEXT I
1270 RETURN
1280 PRINT "JZ"," S(Z)"
1290 FOR I=1 TO N
1300 PRINT USING "-2D.3D10X-2D.3D":X(I),Y(I)
1310 NEXT I
1320 RETURN
1330 REM
1340 DELETE G,G1,G2
1350 DIM G(17)
1360 DATA 0,0,0,0,0,0,0,0,1,0,20,120,10,90,33,32,1
1370 X$="POSITION IN CELL (cm)"
1380 Y$="INTENSITY"
1390 PRINT " ENTER TITLE: "
1400 INPUT T$
1410 RESTORE 1360
1420 READ G
1430 G(1)=N
1440 G(2)=Z1
1450 G(3)=Z2
1460 G(4)=Z7/2
1470 G(6)=3*N0
```

```
1480 G(7)=N0
1490 G(15)=U2
1500 G(10)=D2
1510 PRINT "ENTER PLOTTER UNIT # (32): ";
1520 INPUT G(16)
1530 GOSUB 2010
1540 RETURN
1550 REM MOD EXP FUNCTION
1560 DEF FNE(X)=(ABS(X)<100)*EXP((ABS(X)<100)*X)
1570 S(1)=0
1580 FOR I=1 TO N0
1590 FOR J=1 TO N0
1600 P1(I,J)=P(I)↑2+P(J)↑2
1610 C(I,J)=P(I)*P(J)*I0(I)*I0(J)/P1(I,J)↑0.5
1620 S(1)=S(1)+C(I,J)*FNE(-((D(I)-D(J))↑2/P1(I,J)))
1630 NEXT J
1640 NEXT I
1650 S(1)=S(1)*PI↑0.5
1660 S(2)=4*P4*PI↑0.5*FNE(-2*Z5*Z7)
1670 RETURN
1680 S(3)=FNE(-2*Z5*(Z7+Z))+P4↑2*FNE(-2*Z5*(Z7-Z))
1690 S(4)=0
1700 T=Z/V
1710 FOR I=1 TO N0
1720 S(4)=S(4)+I0(I)↑2*P(I)*FNE(-(2*(T/P(I))↑2))
1730 NEXT I
1740 S(5)=0
1750 IF N0=1 THEN 1820
1760 FOR I=1 TO N0-1
1770 FOR J=I+1 TO N0
1780 S(5)=S(5)+C(I,J)*FNE(-((2*T+(D(I)-D(J)))↑2/P1(I,J)))
1790 S(5)=S(5)+C(I,J)*FNE(-((2*T-(D(I)-D(J)))↑2/P1(I,J)))
1800 NEXT J
1810 NEXT I
1820 S0=N3*(S(3)*S(1)+S(2)*(S(4)/2↑0.5+S(5)))
```

```
1830 RETURN
1840 REM YES RESPONSE
1850 INPUT Q$
1860 Q=POS(Q$,"Y",1)
1870 Q=Q<>0
1880 RETURN
1890 REM HIGHLITE
1900 FOR I=1 TO 25
1910 PRINT " WRITING ON FILE # ";D2;" UNIT # ";U2;"GK"
1920 NEXT I
1930 PRINT "J";
1940 RETURN
1950 PRINT " ENTER Z VALUEG ";
1960 INPUT Z
1970 Q1=0
1980 Q2=0
1990 PAGE
2000 RETURN
2010 REM PLOT
2020 REM
2030 REM
2040 REM GRAPHICS PLOT ALGORITHM APPENDED
2050 REM
2060 REM
2070 RETURN
```

```
1 REM---LASER FWHM FROM TPF GRAPH (1 PULSE)---012/79-03
2 GO TO 100
4 REM RIGHT
5 01=01+03/05
7 RETURN
8 REM LEFT
9 01=01-03/05
11 RETURN
12 REM UP
13 02=02+04/05
15 RETURN
16 REM DOWN
17 02=02-04/05
19 RETURN
20 REM FINE/STD
21 05=1
23 RETURN
24 REM COURSE
25 05=0.1
27 RETURN
28 REM X-FINE
29 05=10
31 RETURN
32 REM XX-FINE
33 05=100
34 RETURN
79 RETURN
80 REM GEN
81 P3=1
82 PRINT "IG"
83 RETURN
99 RETURN
100 GOSUB 200
110 GOSUB 1300
```

```
120 GOSUB 1400
130 GOSUB 280
140 GOSUB 890
150 GOSUB 370
160 GOSUB 650
170 GOSUB 1140
180 GOSUB 980
190 RETURN
200 INIT
210 PAGE
220 DIM P(15),P6(2,2),P7(2,1),P8(2,1)
230 P$=""
240 V=2.22E+10
250 SET KEY
260 O5=1
270 RETURN
280 REM LOCATE BKG
290 PRINT P$;"↑LOCATE AND GIN IN TWO BKG PEAK BOUNDS: G"
300 GOSUB 1040
310 P(1)=O1
320 P(2)=O2
330 GOSUB 1040
340 P(3)=O1
350 P(4)=O2
360 RETURN
370 DIM F1(3,1),F2(3,3),F3(3,1)
380 F1=0
390 F2=0
400 FOR I=1 TO P2
410 IF X(I)>P(1) AND X(I)<P(3) THEN 510
420 X(I)=X(I)-P(11)
430 F1(1,1)=P(12)/3
440 F1(2,1)=F1(2,1)+X(I)*Y(I)
450 F1(3,1)=F1(3,1)+X(I)↑2*Y(I)
460 F2(2,1)=F2(2,1)+X(I)
```

```
470 F2(3,3)=F2(3,3)+X(I)↑4
480 F2(3,1)=F2(3,1)+X(I)↑2
490 F2(2,3)=F2(2,3)+X(I)↑3
500 X(I)=X(I)+P(11)
510 NEXT I
520 F2(1,2)=0
530 F2(1,1)=1
540 F2(1,3)=0
550 F2(2,2)=F2(3,1)
560 F2(3,2)=F2(2,3)
570 F2=INV(F2)
580 F3=F2 MPY F1
590 DEF FNB(X)=F3(1,1)+F3(2,1)*X+F3(3,1)*X↑2
600 MOVE X(1),FNB(X(1))
610 FOR I=1 TO P2
620 DRAW X(I),FNB(X(I))
630 NEXT I
640 RETURN
650 REM LOC PEAK BOUNDS
660 P4=0.2
670 PRINT "↑JJJ"
680 FOR I=1 TO P2
690 IF P(11)<X(I) THEN 730
700 NEXT I
710 PRINT "X-COORDINATE OF PEAK NOT FOUND"
720 STOP
730 X0=I
740 FOR I=X0 TO P2
750 IF Y(I)-FNB(X(I))<P4*(P(12)-FNB(P(11))) THEN 790
760 NEXT I
770 PRINT "RIGHT PEAK BOUND NOT FOUND"
780 STOP
790 P(9)=X(I)
800 P(10)=Y(I)
810 FOR I=X0 TO 1 STEP -1
```

```
820 IF Y(I)-FNB(X(I))<P4*(P(12)-FNB(P(11))) THEN 860
830 NEXT I
840 PRINT "LEFT PEAK BOUND NOT FOUND"
850 STOP
860 P(7)=X(I)
870 P(8)=Y(I)
880 RETURN
890 REM LOC PEAK
900 HOME
910 PRINT P$;"JLOCATE AND GIN IN PEAK MAX: G"
920 O1=X(P2/2)
930 O2=Y(P2/2)
940 GOSUB 1040
950 P(11)=O1
960 P(12)=O2
970 RETURN
980 REM PRI OUTPUT
990 PRI USI 1000:"IJJJFWHM = ",SQR(8*LOG(2)/V↑2/ABS(P7(2,1)))*1.0E+12
1000 IMAGE 11A2D.2D" PSEC "S
1010 PRI USI 1020:"P/B(0) =",P(12)/FNB(P(11)),"P/B(d) =",P(12)/FNB(X(1))
1020 IMAGE 1X08A2D.2D1X08A2D.2D
1030 RETURN
1040 REM GIN
1050 MOVE O1,O2
1060 PRINT @32,18:5
1070 P3=0
1080 MOVE O1,O2
1090 PRINT @32,24:"|"
1100 IF P3 THEN 1120
1110 GO TO 1080
1120 PRINT @32,18:0
1130 RETURN
1140 REM CURVEFIT
1150 P6=0
1160 P7=0
```

```
1170 P8=0
1180 FOR P1=1 TO P2
1190 IF X(P1)<P(7) MIN P(9) OR X(P1)>P(7) MAX P(9) THEN 1270
1200 P3=Y(P1)-FNB(X(P1)-P(11))
1210 P3=LOG(P3 MAX 1.0E-64)
1220 P6(1,2)=P6(1,2)+(X(P1)-P(11))↑2
1230 P6(2,2)=P6(2,2)+(X(P1)-P(11))↑4
1240 P8(1,1)=P8(1,1)+P3
1250 P8(2,1)=P8(2,1)+P3*(X(P1)-P(11))↑2
1260 P6(1,1)=P6(1,1)+1
1270 NEXT P1
1280 P1=P6(1,1)
1290 P6(2,1)=P6(1,2)
1300 P6=INV(P6)
1310 P7=P6 MPY P8
1320 DEF FNF(X)=EXP(P7(1,1)+P7(2,1)*(X-P(11))↑2)+FNB(X-P(11))
1330 MOVE P(7) MIN P(9),FNF(P(7) MIN P(9))
1340 FOR P3=P(7) MIN P(9) TO P(7) MAX P(9) STEP ABS(P(7)-P(9))/P1
1350 DRAW P3,FNF(P3)
1360 NEXT P3
1370 RETURN
1380 REM DATA INPUT
1390 PRINT "INSERT DATA TAPE AND ENTER FILE #:GG ";
1400 INPUT P3
1410 FIND P3
1420 READ @33:P2
1430 DIM X(P2),Y(P2)
1440 FOR I=1 TO P2
1450 READ @33:X(I),Y(I)
1460 NEXT I
1470 RETURN
1480 REM PLOT
1490 DATA 0,-0.85,0.85,0.85,0,0,1,3,0,0,20,120,20,80,33,32,1
1500 RESTORE 1490
1510 DIM G(17)
```

```
1520 READ G
1530 G(1)=P2
1540 G(6)=0
1550 FOR P1=1 TO P2
1560 G(6)=G(6) MAX Y(P1)
1570 NEXT P1
1580 G(6)=INT(G(6)+1.4)
1590 X$="POSITION IN CELL (cm)"
1600 Y$="INTENSITY"
1610 T$="HHHHHHHHHHHHHHHFWHM & P/B"
1620 C$="+"
1630 O3=(G(3)-G(2))/100
1640 O4=(G(6)-G(5))/100
1650 G(10)=0
1660 O1=G(2)
1670 O2=G(5)
1680 GOSUB 1700
1690 RETURN
1700 REM PLOT
1710 REM
1720 REM
1730 REM GRAPHICS PLOT ALGORITHM APPENDED
1740 REM
1750 REM
1760 RETURN
```

```
1 REM---GRAPHICS PLOT ALGORITHM "SUBROUTINE"---05/79-03
30001 REM G(I)==1-#PTS 2-XMIN 3-XMAX 4-XSTP 5-YMIN 6-YMAX 7-YSTP
30002 REM 8-(0)LINE,(1)CHAR,(2)LIN/CAR,(3)NULLPTS 9-(0)NOGRID,(1)GRID
30003 REM 10-DAT FILE # 11,12,13,14-VIE(1,2,3,4) 15-READ UNIT
30004 REM 16-PLOT UNIT 17-(0)VERT YTITLE,(1)ROT YTITLE
30005 REM NEED X$,Y$,T$,C$ AND DATA X,Y(OR FILE) DUMMY:G1,G2,W$(1)
30006 REM ***START***
30007 REM REQ DATA TAPE
30008 GOSUB 30115
30009 REM PAGE INIT
30010 GOSUB 30169
30011 REM SET VIEWPORT
30012 GOSUB 30094
30013 REM SET WINDOW
30014 GOSUB 30096
30015 REM CALL AXIS OR GRID
30016 GOSUB 30181
30017 REM FIN DATA FILE & READ # PTS
30018 GOSUB 30111
30019 REM CALL PLT X,Y:X,Y,C$:X,Y,C$/LINE:X,Y,NULL PTS
30020 REM CALL PLT DATFILE:DATFIL,C$:DATFIL,C$/LIN:DATFIL,NULL PTS
30021 GOSUB 30178
30022 REM CALL BOX
30023 GOSUB 30172
30024 REM CALL LABLE X
30025 GOSUB 30044
30026 REM CALL LABLE Y
30027 GOSUB 30034
30028 REM CALL TITLES
30029 GOSUB 30055
30030 REM CALL HOME
30031 GOSUB 30085
30032 RETURN
30033 STOP
```

```
30034 FOR G1=G(5) TO G(6) STEP G(7)
30035 MOVE @G(16):G(2),G1
30036 IF ABS(G1)<0.1 OR ABS(G1)>99 AND G1<>0 THEN 30040
30037 PRINT @G(16):"HHHHHHH";
30038 PRINT @G(16): USING "-3D.2D":G1
30039 GO TO 30042
30040 PRINT @G(16):"HHHHHHHHH";
30041 PRINT @G(16): USING "1E":G1
30042 NEXT G1
30043 RETURN
30044 FOR G1=G(2) TO G(3) STEP 2*G(4)
30045 MOVE @G(16):G1,G(5)
30046 IF ABS(G1)<0.1 OR ABS(G1)>99 AND G1<>0 THEN 30050
30047 PRINT @G(16):"HHHJ";
30048 PRINT @G(16): USING "-3D.2D":G1
30049 GO TO 30052
30050 PRINT @G(16):"HHHJ";
30051 PRINT @G(16): USING "1E":G1
30052 NEXT G1
30053 PRINT @G(16):
30054 RETURN
30055 MOVE @G(16):(G(2)+G(3))/2,G(5)
30056 PRINT @G(16):"JJJ";
30057 FOR G1=1 TO LEN(X$)/2
30058 PRINT @G(16):"H";
30059 NEXT G1
30060 PRINT @G(16):X$
30061 MOVE @G(16):(G(2)+G(3))/2,G(6)
30062 PRINT @G(16):"K";
30063 FOR G1=1 TO LEN(T$)/2
30064 PRINT @G(16):"H";
30065 NEXT G1
30066 PRINT @G(16):T$
30067 MOVE @G(16):G(2),(G(5)+G(6))/2
30068 PRINT @G(16):"HHHHHHHHHHH";
```

```
30069 IF G(17)=1 AND G(16)<>32 THEN 30078
30070 FOR G1=1 TO LEN(Y$)/2
30071 PRINT @G(16):"K";
30072 NEXT G1
30073 FOR G1=1 TO LEN(Y$)
30074 W$=SEG(Y$,G1,1)
30075 PRINT @G(16):W$;"HJ";
30076 NEXT G1
30077 RETURN
30078 FOR G1=1 TO LEN(Y$)/2
30079 PRINT @G(16):"J";
30080 NEXT G1
30081 PRINT @G(16),25:90
30082 PRINT @G(16):"J";Y$
30083 PRINT @G(16),25:0
30084 RETURN
30085 HOME @G(16):
30086 PRINT @G(16):"GGGG";
30087 PRINT @32,26:0
30088 RETURN
30089 SCALE 1,1
30090 RMOVE @G(16):-0.5*1.55,-0.5*1.88
30091 PRINT @G(16):C$;
30092 WINDOW G(2),G(3),G(5),G(6)
30093 RETURN
30094 VIEWPORT G(11),G(12),G(13),G(14)
30095 RETURN
30096 WINDOW G(2),G(3),G(5),G(6)
30097 RETURN
30098 G1=(SGN(G(2))=SGN(G(3)))*G(2)
30099 G2=(SGN(G(5))=SGN(G(6)))*G(5)
30100 AXIS @G(16):G(4),G(7),G1,G2
30101 RETURN
30102 FOR G1=G(2) TO G(3) STEP G(4)
30103 MOVE @G(16):G1,G(5)
```

```
30104 DRAW @G(16):G1,G(6)
30105 NEXT G1
30106 FOR G1=G(5) TO G(6) STEP G(7)
30107 MOVE @G(16):G(2),G1
30108 DRAW @G(16):G(3),G1
30109 NEXT G1
30110 RETURN
30111 IF G(10)=0 THEN 30114
30112 PRINT @G(15),27:G(10)
30113 READ @G(15):G(1)
30114 RETURN
30115 IF G(10)=0 THEN 30118
30116 PRINT "INIT DATA TAPE ON UNIT ";G(15);" AND HIT RETURNGG ";
30117 INPUT W$
30118 RETURN
30119 MOVE @G(16):X(1),Y(1)
30120 DRAW @G(16):X,Y
30121 RETURN
30122 FOR G1=1 TO G(1)
30123 MOVE @G(16):X(G1),Y(G1)
30124 GOSUB 30089
30125 NEXT G1
30126 RETURN
30127 FOR G1=1 TO G(1)
30128 MOVE @G(16):X(G1),Y(G1)
30129 DRAW @G(16):X(G1),Y(G1)
30130 NEXT G1
30131 RETURN
30132 MOVE @G(16):X(1),Y(1)
30133 FOR G1=1 TO G(1)
30134 DRAW @G(16):X(G1),Y(G1)
30135 GOSUB 30089
30136 MOVE @G(16):X(G1),Y(G1)
30137 NEXT G1
30138 RETURN
```

```
30139 READ @G(15):G2,G3
30140 MOVE @G(16):G2,G3
30141 FOR G1=2 TO G(1)
30142 READ @G(15):G2,G3
30143 DRAW @G(16):G2,G3
30144 NEXT G1
30145 RETURN
30146 FOR G1=1 TO G(1)
30147 READ @G(15):G2,G3
30148 MOVE @G(16):G2,G3
30149 GOSUB 30089
30150 NEXT G1
30151 RETURN
30152 FOR G1=1 TO G(1)
30153 READ @G(15):G2,G3
30154 MOVE @G(16):G2,G3
30155 DRAW @G(16):G2,G3
30156 NEXT G1
30157 RETURN
30158 READ @G(15):G1,G2
30159 MOVE @G(16):G1,G2
30160 GOSUB 30089
30161 MOVE @G(16):G1,G2
30162 FOR G1=2 TO G(1)
30163 READ @G(15):G2,G3
30164 DRAW @G(16):G2,G3
30165 GOSUB 30089
30166 MOVE @G(16):G2,G3
30167 NEXT G1
30168 RETURN
30169 PAGE @G(16):
30170 PRINT @32,26:1
30171 RETURN
30172 MOVE @G(16):G(2),G(5)
30173 DRAW @G(16):G(3),G(5)
```

```
30174 DRAW @G(16):G(3),G(6)
30175 DRAW @G(16):G(2),G(6)
30176 DRAW @G(16):G(2),G(5)
30177 RETURN
30178 G1=1+G(8)+4*(G(10)<>0)
30179 GOSUB G1 OF 30119,30122,30132,30127,30139,30146,30158,30152
30180 RETURN
30181 GOSUB 1+G(9) OF 30098,30102
30182 RETURN
30183 STOP
```

REFERENCES

1. J. A. Giordmaine, P. M. Rentzepis, S. L. Shapiro, and K. W. Wecht, Appl. Phys. Lett. 11, 216(1967).

2. J. Taboada and D. D. Venable, J. Appl. Phys. 49, 5669(1978).

3. This expression is valid for the Guassian pulse outside of the dye cell where $\alpha=0$ and $\beta=0$. For a discussion of the Gaussian function, see, for example, Bevinton, R., Data Reduction and Error Analysis for for the Physics Sciences, (McGraw-Hill, New York, 1969), chapter 3.

4. The numerical integration algorithm was based on the Simpson's Rule. See for example, Sokolnikoff, I.S. and Redheffer, R.M., Mathematics of Physics and Modern Engineering, (McGraw-Hill, New York, 1966), chapter 10.

5. D. D. Venable and J. Taboada, J. Appl. Phys., 50, 5996(1979).